Grade Two

In previous grades students gained an understanding of whole numbers to 120 and developed strategies to add, subtract, and compare numbers. They solved addition and subtraction word problems within 20 and developed fluency with these operations within 10. They developed a foundation for understanding place value, including grouping in tens and ones. They also worked with non-standard measurement and reasoned about attributes of geometric shapes (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

WHAT STUDENTS LEARN IN GRADE TWO

[Note: Sidebar]

Grade Two Critical Areas of Instruction

In grade two, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. (CCSSO 2010, Grade 2 Introduction).

Students also work towards fluency with addition and subtraction within 100 and they know from memory all sums of two one-digit numbers.

Grade Two Standards for Mathematical Content

The Standards for Mathematical Content emphasize key content, skills, and practices at each grade level and support three major principles:

- **Focus:** Instruction is focused on grade level standards.
- **Coherence:** Instruction should be attentive to learning across grades and linking major topics within grades.
- **Rigor:** Instruction should develop conceptual understanding, procedural skill and fluency, and application.

Grade level examples of focus, coherence, and rigor will be indicated throughout the chapter.

Not all of the content in a given grade is emphasized equally in the standards. Cluster headings can be viewed as the most effective way to communicate the **focus** and **coherence** of the standards. Some clusters of standards require a greater instructional emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the later demands of college and career readiness.

The following Grade 2 Cluster-Level Emphases chart highlights the content emphases in the standards at the cluster level for this grade. The bulk of instructional time should be given to “Major” clusters and the standards within them. However, standards in the “Supporting” and “Additional” clusters should not be neglected. To do so will result in gaps in students’ learning, including skills and understandings they may need in later grades. Instruction should reinforce topics in major clusters by utilizing topics in the supporting and additional clusters. Instruction should include problems and activities that support natural connections between clusters.

Teachers and administrators alike should note that the standards are not topics to be checked off a list during isolated units of instruction, but rather content to be developed throughout the school year through rich instructional experiences and presented in a coherent manner (Adapted from the Partnership for Assessment of Readiness for College and Careers [PARCC] 2012).

[**Note**: The Emphases chart should be a graphic inserted in the grade level section. The explanation “key” needs to accompany it.]

Grade 2 Cluster-Level Emphases

Operations and Algebraic Thinking

- [m]: Represent and solve problems involving addition and subtraction. **(2.OA.1▲)**
- [m]: Add and subtract within 20. **(2.OA.2▲)**
- [a/s]: Work with equal groups of objects to gain foundations for multiplication. **(2.OA.3-4)**

Number and Operations in Base Ten

- [m]: Understand place value. **(2.NBT.1-4▲)**
- [m]: Use place value understanding and properties of operations to add and subtract. **(2.NBT.5-9▲)**

Measurement and Data

- [m]: Measure and estimate lengths in standard units. **(2.MD.1-4▲)**
- [m]: Relate addition and subtraction to length. **(2.MD.5-6▲)**
- [a/s]: Work with time and money. **(2.MD.7-8)**
- [a/s]: Represent and interpret data. **(2.MD.9-10)**

Geometry

- [a/s]: Reason with shapes and their attributes. (**2.G.1-3**)

Explanations of Major, Additional and Supporting Cluster-Level Emphases
Major[1] [m] (▲) clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.
Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade **Supporting [s]** clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis. *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

(Adapted from Achieve the Core 2012)

Connecting Mathematical Practices and Content

The Standards for Mathematical Practice (MP) are developed throughout each grade and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject that makes use of their ability to make sense of mathematics. The MP standards represent a picture of what it

[1] The ▲symbol will indicate standards in a Major Cluster in the narrative.

looks like for students to understand and do mathematics in the classroom and should be integrated into every mathematics lesson for all students.

Although the description of the MP standards remains the same at all grades, the way these standards look as students engage with and master new and more advanced mathematical ideas does change. Below are some examples of how the MP standards may be integrated into tasks appropriate for Grade 2 students. (Refer to pages 9–12 in the Overview of the Standards Chapters for a complete description of the MP standards.)

Standards for Mathematic Practice (MP)

Explanations and Examples for Grade Two

Standards for Mathematical Practice	Explanation and Examples
MP.1. Make sense of problems and persevere in solving them.	In second grade, students realize that doing mathematics involves reasoning about and solving problems. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They make conjectures about the solution and plan out a problem-solving approach.
MP.2. Reason abstractly and quantitatively.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Students represent situations by decontextualizing tasks into numbers and symbols. For example, in the task, “There are 25 children in the cafeteria, and they are joined by 17 more children. How many students are in the cafeteria? ” Students translate the situation into an equation, such as: 25 + 17 = __ and then solve the problem. Students also contextualize situations during the problem solving process. Teachers might ask, “How do you know” or “What is the relationship of the quantities?” to reinforce students’ reasoning and understanding.
MP.3. Construct viable arguments and critique the reasoning of others.	Second graders may construct arguments using concrete referents, such as objects, pictures, math drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” “Explain your thinking,” and “Why is that true?” They not only explain their own thinking, but also listen to others’ explanations. They decide if the explanations make sense and ask appropriate questions. Students critique the strategies and reasoning of their classmates. For example, to solve 74 – 18, students might use a variety of strategies and discuss and critique each other’s

	reasoning and strategies.
MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including writing numbers, using words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations. Students need opportunities to connect the different representations and explain the connections. Students model real-life mathematical situations with an equation and check to make sure that their equation accurately matches the problem context. They use concrete manipulative and/or math drawings to explain the equation. They create an appropriate problem situation from an equation. For example, students create a story problem for the equation 43 + □ = 82 such as “There were 43 gumballs in the machine. Tom poured in some more gumballs. There are 82 gumballs in the machine now. How many did Tom pour in?” Students should be encouraged to answer questions, such as “What math drawing or diagram could you make and label to represent the problem?” or “What are some ways to represent the quantities?”
MP.5. Use appropriate tools strategically.	In second grade, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited than others. For instance, second graders may decide to solve a problem by making a math drawing rather than writing an equation. Students may use tools such as snap cubes, place value (base ten) blocks, hundreds number boards, number lines, rulers, virtual manipulatives, diagrams, and concrete geometric shapes (e.g., pattern blocks, three-dimensional solids). Students understand which tools are the most appropriate to use. For example, while measuring the length of the hallway, students can explain why a yardstick is more appropriate to use than a ruler. Students should be included to answer questions such as, “Why was it helpful to use…?”
MP.6. Attend to precision.	As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning. Students communicate clearly, using grade-level appropriate vocabulary accurately and precise explanations and reasoning to explain their process and solutions. For example, when measuring an object, students carefully line up the tool correctly to get an accurate measurement. During tasks involving number sense, students consider if their answer is reasonable and check their work to ensure the accuracy of solutions.
MP.7. Look for and make use of structure.	Second graders look for patterns and structures in the number system. For example, students notice number patterns within the tens place as they connect counting by 10s to corresponding numbers on a 100s chart. Students see structure in the base-ten number system as they understand that 10 ones equal a ten, and 10 tens equal a hundred. Teachers might ask, “What do you notice when…?” or “How do you know if something is a pattern?” Students adopt mental math strategies based on patterns (making ten, fact families, doubles). They use structure to understand subtraction as an unknown addend problem (e.g., 50 – 33 = __ can be written as 33 + __ = 50 and can be thought of as “How much more do I need to add to 33 to get to 50?”).
MP.8. Look for and express regularity in repeated reasoning.	Second grade students notice repetitive actions in counting and computation (e.g., number patterns to count by tens or hundreds). Students continually check for the reasonableness of their solutions during and after completing a task by asking themselves, “Does this make sense?” Students should be encouraged to answer

	questions, such as “What is happening in this situation?” or “What predictions or generalizations can this pattern support?”

(Adapted from Arizona Department of Education [Arizona] 2012 and North Carolina Department of Public Instruction [N. Carolina] 2011)

Standards-based Learning at Grade Two

The following narrative is organized by the domains in the Standards for Mathematical Content and highlights some necessary foundational skills from previous grades and provides exemplars to explain the content standards, highlight connections to the various Standards for Mathematical Practice (**MP**), and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. A triangle symbol (**▲)** indicates standards in the major clusters (refer to the Grade 2 Cluster-Level Emphases table on page 2).

Domain: Operations and Algebraic Thinking

In first grade students solved addition and subtraction word problems within 20 and developed fluency with these operations within 10. A critical area of instruction in grade two is building fluency with addition and subtraction. Second grade students fluently add and subtract within 20 and solve addition and subtraction word problems involving all unknowns within 100. Second graders also work with equal groups of objects to gain the foundations for multiplication.

Operations and Algebraic Thinking	2.OA
Represent and solve problems involving addition and subtraction. 1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions*,* e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]	

In grade two students add and subtract numbers within 100 in the context of one- and two-step word problems (**2.OA.1▲**). By second grade students have had prior

[1] See Glossary, Table 1.

experiences working with various problem situations (add to, take from, put together, take apart, and compare) with unknowns in all positions (result unknown, change unknown, and start unknown). Grade two students extend their work with addition and subtraction word problems in two major ways:

- They represent and solve problems of all types which involve addition and subtraction within 100, building upon their previous work within 20, and
- They represent and solve two-step word problems of all types, extending their work with one-step word problems.

(Adapted from Arizona 2012, N. Carolina 2013, Georgia Department of Education [Georgia] 2011, and the Kansas Association of Teachers of Mathematics [KATM] 2nd FlipBook 2012)

The various addition and subtraction problem types are listed in the following table.

Grade Two Addition and Subtraction Problem Types.
Add To (with change unknown): "Bill had 25 baseball cards. His mom gave him some more. Now he has 73 baseball cards. How many baseball cards did his mom give him?"
In this problem the starting quantity is provided (25 baseball cards), a second quantity is added to that amount (some baseball cards) and the result quantity is given (73 baseball cards). This question type is more algebraic and challenging than the "result unknown" problems and can be modeled by a situational equation 25 + □ = 73, which does not immediately lead to the answer. Students can write a related equation, 73 – 25 = □ (called a solution equation) to solve the problem.
Take From (with change unknown): **"**Andrea had 51 stickers. She gave some stickers away. Now she has 22 stickers. How many stickers did she give away?"
This question can be modeled by a situational equation 51 − □ = 22 or a solution equation 51 – 22 = □. Both the Take From and Add To questions involve actions.
Add to (with start unknown): Some children were playing in the playground. 5 more children joined them. Then there were 22 children. How many children were playing before?"
This problem can be represented by □ + 5 = 22. The "start unknown" problems are difficult for students to model because the initial quantity is unknown and therefore some students do not know how to start a solution strategy. They can make a drawing, where it is key that they realize that the 5 is part of the 22 total children. This leads to more general solutions by subtracting the known addend or counting/adding on from the known addend to the total.
Take From (with start unknown): Some children were lining up for lunch. 4 children left and then there were 26 children still waiting in line. How many children were there before?
This problem can be modeled by □ – 4 = 26. Similar to the previous Add To (start unknown) problem, the Take From problems with the "start unknown" require a high level of conceptual understanding. Students need to understand that the total is first in a subtraction equation, and that this total is broken apart into the 4 and the 26.
Put Together/Take Apart (with addend unknown): Roger puts 24 apples in a fruit basket. 9 are red

and the rest are green. How many are green?"
There is no direct or implied action. The problem involves a set and its subsets. It can be modeled by 24 – 9 = □ or 9 + □ = 24. This type of problem provides students with opportunities to understand subtraction as an unknown-addend problem.
Compare (with difference unknown): Pat has 19 peaches. Lynda has 14 peaches. How many more peaches does Pat have than Lynda?"
Compare problems involve relationships between quantities. While most adults might use subtraction to solve this type of Compare problem (19 – 14 = □), students will often solve this problem as an unknown addend problem (14 + □ = 19) by using a counting up or matching strategy. In all mathematical problem solving, what matters is the explanation a student gives to relate a representation to a context, and not the representation separated from its context.
Compare (with bigger unknown--"more" version): Theo has 23 action figures. Rosa has 2 more action figures than Theo. How many action figures does Rosa have?"
This problem can be modeled by 23 + 2 = □.
Compare (with bigger unknown--"fewer" version): Lucy has 28 apples. She has 2 fewer apples than Marcus. How many apples does Marcus have?"
This problem can be modeled as 28 + 2 = □. The misleading language form "fewer" may lead students to choose the wrong operation.
Compare (with smaller unknown--"fewer" version): Bill has 24 stamps. Lisa has 2 fewer stamps than Bill. How many stamps does Lisa have?"
This problem can be modeled as 24 – 2 = □.
Compare (with smaller unknown--"more" version): David has 27 more bunnies than Keisha. David has 28 bunnies. How many bunnies does Keisha have?"
This problem can be modeled by 28 – 27 = □. The misleading language form "more" may lead students to choose the wrong operation.

[Note: Also refer to Table 1 "Common Addition and Subtraction Situations" in the Glossary.]

For these more complex grade two problems, it is important for students to represent the problem situations with drawings and equations (**2.OA.1▲**). Drawings can be shown more easily to the whole class during explanations and can be related to equations. Students can also use manipulatives (e.g., snap cubes, place-value blocks) but making drawings of quantities can be used anywhere to solve problems and support students in describing their strategies. Second grade students represent problems with equations and use boxes, blanks, or pictures for the unknown amount. For example, students can represent compare problems using "comparison bars" (e.g., a long bar above, a shorter bar below, followed by an oval for the difference or unknown amount, where the shorter bar plus the oval are the same length as the longer bar on top). Students can draw these bars and fill in numbers from the problem and label the bars.

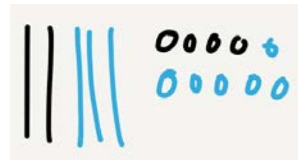
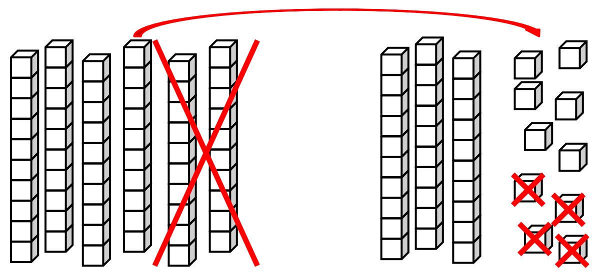
One-step word problems use one operation. New at second grade are two-step word problems (**2.OA.1▲**) that require students to complete two operations, which may include the same operation or opposite operations.

Initially two-step problems should not involve the most difficult subtypes of problems (e.g., compare and start unknown problems) and should be limited to only single-digit addends. There are many problem situation subtypes and various ways to combine such subtypes to devise two-step problems. Introducing easier problems first will provide support for second grade students who are still developing proficiency with the compare and start unknown problems. (Adapted from Progressions K-5 OA 2011)

The following table has examples of easy and middle-difficulty two-step word problems that would be appropriate.

One-Step Word Problem One Operation	**Two-Step Word Problem** Two Operations, Same	**Two-Step Word Problem** Two Operations, Opposite
There are 15 stickers on the page. Brittany put some more stickers on the page and now there are 22. How many stickers did Brittany put on the page? 15 + __ = 22 22 – 15 = __	There are 9 blue marbles and 6 red marbles in the bag. Maria put in 8 more marbles. How many marbles are in the bag now? 9 + 6 + 8 = __ or (9 + 6) + 8 = __	There are 39 peas on the plate. Carlos ate 25 peas. Mother put 7 more peas on the plate. How many peas are on the plate now? 39 – 25 + 7 = or (39 – 25) + 7 = __

Second graders use a range of methods, often mastering more complex strategies such as making tens and doubles and near doubles that were introduced in grade one for problems involving single-digit addition and subtraction. Second grade students also begin to apply their understanding of place value to solve problems:

One-Step Problem: Some students are in the cafeteria. 24 more students came in. Now there are 60 students in the cafeteria. How many were in the cafeteria to start with? Use drawings and equations to show your thinking.
Student A: I read the problem and thought about how to write it with numbers. I thought, “What and 24 makes 60?” I used a math drawing to solve it. I started with 24. Then I added tens until I got close to 60; I added 3 tens. I stopped at 54. Then, I added 6 more ones to get to 60. So, 10 + 10 + 10 + 6 = 36. So, there were 36 students in the cafeteria to start with. My equation for the problem is □ + 24 = 60. **(MP.2, MP.7 and MP.8)** 
Student B: I read the problem and thought about how to write it with numbers. I thought, “There are 60 total. I know about the 24. So, what is 60 – 24?” I used place value blocks to solve it. I started with 60 and took 2 tens away. I needed to take 4 more away. So, I broke up a ten into ten ones. Then, I took 4 away. That left me with 36. So, 36 students were in the cafeteria at the beginning. 60 – 24 = 36. My equation for the problem is 60 – 24 = □. **(MP.2, MP.4, MP.5 and MP.6)** 

(Adapted from Arizona 2012, N. Carolina 2013, Georgia 2011, and KATM 2nd FlipBook 2012)

As students solve addition and subtraction word problems they use concrete manipulatives, pictorial representations, and mental mathematics to make sense of a problem (**MP.1**); they reason abstractly and quantitatively as they translate word problem situations into equations (**MP.2**); and they model with mathematics (**MP.4**).

Following is a sample classroom activity that connects the Standards for Mathematical Content and Standards for Mathematical Practice.

Connecting to the Standards for Mathematical Practice—Grade 2

Standard(s) Addressed	***Example(s) and Explanations***
2.OA.1: Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem. **2.NBT.5:** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	**Task:** ***Base-10 Block Activities*.** This is a two-tiered approach to problem solving with basic operations within 100. The first task involves students seeing various strategies for adding two-digit numbers using base-10 blocks. The second is an extension that builds facility in adding and subtracting such numbers. 1. Teachers should present several problem situations that involve addition and subtraction in which students can use base-ten blocks to model their solution strategies. Such solutions are made public via an overhead display or by the teacher rephrasing and demonstrating student solutions. Such problems might include: • *Micah had 24 marbles while Sheila had 15. They decided to put all of their marbles in a box. How many marbles were there altogether?* (This is an addition problem that does not require bundling ones into a ten.) • *There were 28 boys and 35 girls on the playground at recess. How many children were there on the playground at recess?* (This is an addition problem that requires bundling.) • *There were 48 cows on a pasture. 17 of the cows went into the barn. How many cows are left on the pasture?* (This is a subtraction problem that does not require exchanging a ten for ones.) • *There were 54 candies in a bowl. 26 students were allowed to take one candy each. How many candies are left over after the children have taken theirs?* (This is subtraction involving exchanging a ten for 10 ones.) 2. Next, the teacher can play a game that reinforces understanding of addition and subtraction and skill in doing addition and subtraction. Each student takes out base-10 blocks to represent a given number, say 45. The teacher then asks students how many more blocks are needed to make 80. Students represent the difference with base-10 blocks and justify how they know their answers are correct. The teacher can ask several variations of this same basic question; the task can be used repeatedly throughout the school year to reinforce concepts of operations. **Classroom Connections:** When students are given the opportunity to construct their own strategies for adding and subtracting numbers, they reinforce their understanding of place value and the base-10 number system. Activities such as the one presented here help build this foundation in context and through modeling numbers with objects, such as base-10 blocks. **Connecting to the Standards for Mathematical Practice:** (MP.1) Students are challenged to think through how they would solve a potentially unfamiliar problem situation and to devise a strategy. The teacher can assess where each student is starting from and move him or her forward from there. (MP.3) When students are asked to explain how they solved the problems to their peers, they are essentially constructing a mathematical argument that justifies that they have performed the addition or subtraction correctly. (MP.7) When students begin exchanging sticks and units to represent grouping and breaking apart tens and ones, they are making use of the structure of the base-10 number system to understand addition and subtraction.

Operations and Algebraic Thinking 2.OA

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

In grade two students extend their fluency with addition and subtraction from within 10 to within 20 (**2.OA.2▲**). The extended experiences students have had with addition and subtraction in kindergarten (within 5) and grade one (within 10) culminate in grade two students becoming fluent in single-digit additions and related subtractions using mental Level 2 and 3 methods and strategies as needed.

[Note: Sidebar]

To solve word problems, students learn to apply various computational methods. Kindergarten students generally use Level 1 methods and Level 2 and 3 methods are used in grades one and two.

Methods used for solving single-digit addition and subtraction problems

Level 1: Direct Modeling by Counting All or Taking Away
Represent situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.

Level 2: Counting On
Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is use to monitor the count.
Methods used to find the total or an addend, depending on what is monitored.

Level 3: Convert to an Easier Problem
Decompose an addend and compose a part with another addend.

Refer to Appendix F for additional information about methods used for solving single-digit addition and subtraction problems.

(Adapted from the University of Arizona Progressions Documents for the Common Core Math Standards [Progressions], K-5 CC and OA (pg. 12) 2011).

[2] See standard 1.OA.6 for a list of mental strategies.

Students may still need to support the development of their fluency with math drawings when solving problems. *Math drawings* represent the number of objects counted (using dots and sticks) and do not need to represent the context of the problem. Thinking about numbers using frames of 10 or making drawings using 5-groups and tens can be a helpful way to understand single-digit additions and subtractions. An example of interactive games students can play to develop counting and addition skills are available at http://illuminations.nctm.org/ActivityDetail.aspx?ID=75 (National Council of Teachers of Mathematics [NCTM] Illuminations).

[Note: Sidebar]

FLUENCY

In the standards for kindergarten through grade six there are individual content standards that set expectations for fluency in computation (e.g., "fluently" add and subtract within 20, standard **2.OA.1▲**.) Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary.

The word "fluent" is used in the standards to mean "reasonably fast and accurate" and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.

Mental strategies help students develop fluency as they make sense of number relationships while they add and subtract within 20.

Mental strategies

- Counting on
- Making tens (9 + 7 = (9 + 1) + 6 = 10 + 6)
- Decomposing a number leading to a ten (14 – 6 = 14 – 4 – 2 = 10 – 2 = 8)
- Fact families (8 + 5 = 13 and 13 - 8 = 5)
- Doubles (1 + 1, 2 + 2, 3 + 3, etc.)
- Doubles plus one (7 + 8 = 7 + 7 + 1)

- Relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4)
- Equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 =12 + 1 =13)

Operations and Algebraic Thinking 2.OA

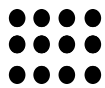
Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Grade three students gain important foundations for multiplication as they explore odd and even numbers in a variety of ways **(2.OA.3**). They use concrete objects (e.g., counters, place-value cubes, etc.) and move towards pictorial representations such as circles or arrays (**MP.1**). Through investigations students realize an even number of objects can be separated into two equal groups (without extra objects remaining), while an odd number of objects will have one object remaining (**MP.7 and MP.8**).

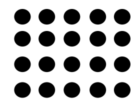
Students also apply their work with doubles addition facts and decomposing (breaking apart) numbers into two equal addends (e.g., 10 = 5 + 5) to understand the concept of even numbers. Students reinforce this concept as they write equations representing sums of two equal addends, such as 2 + 2 = 4, 3 + 3 = 6, 5 + 5 = 10, 6 + 6 = 12, or 8 + 8 =16. Students are encouraged to explain how they determined if a number is odd or even and what strategies they used. (**MP.3**)

With standard **2.OA.4**, second grade students use rectangular arrays to work with repeated addition, a building block for multiplication in grade three, using concrete objects (e.g., counters, buttons, square tiles) as well as pictorial representations on grid paper or other drawings of arrays (**MP.1**). Based on the commutative property of multiplication, students add either the rows or the columns and arrive at the same solution (**MP.2**). Students write equations that represent the total as the sum of equal addends as shown in the following example.



$4 + 4 + 4 = 12$

$3 + 3 + 3 + 3 = 12$



$5 + 5 + 5 + 5 = 20$

$4 + 4 + 4 + 4 + 4 = 20$

The first example will support student understanding that $3 \times 4 = 4 \times 3$, while the second example supports the fact that $4 \times 5 = 5 \times 4$.

(Adapted from Arizona 2012, N. Carolina 2013, Georgia 2011, and KATM 2nd FlipBook 2012)

[Note: Sidebar]

Focus, Coherence, and Rigor:

Student work in this cluster reinforces addition skills and understandings and is connected to major work in the earlier clusters “Represent and solve problems involving addition and subtraction” and “Add and subtract within 20.” (**2.OA.1-2▲**). Also, as students work with odd and even groups (**2.OA.3**) they build a conceptual understanding of equal groups, which supports their introduction to multiplication and division in grade 3.

Domain: Number and Operations in Base Ten

In first grade, students viewed two-digit numbers as amounts of tens and ones. A critical area of instruction in grade two is to extend students’ understanding of base-ten notation. Second grade students understand multi-digit numbers (up to 1000). They add and subtract within 1000 and become fluent with addition and subtraction within 100. (Adapted from Progressions K-5 NBT 2011).

Number and Operations in Base Ten	2.NBT
Understand place value.	

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
    a. 100 can be thought of as a bundle of ten tens — called a "hundred."
    b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by **2s**, 5s, 10s, and 100s. **CA**
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

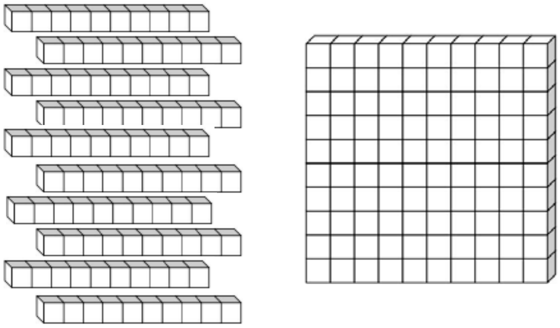
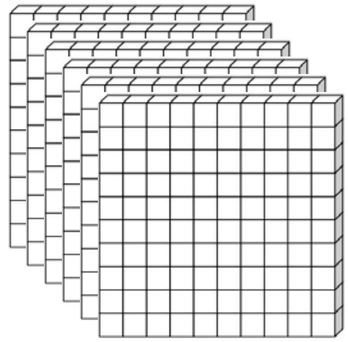
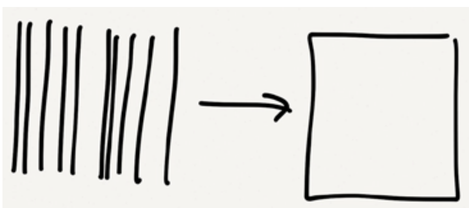
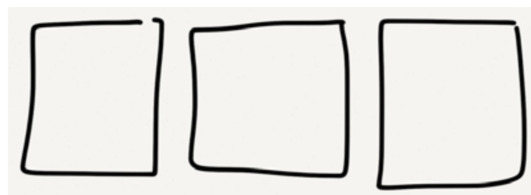
Second grade students understand that the digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones (**2.NBT.1▲**). They understand the following as special cases:

a. 100 can be thought of as a bundle of ten "tens" — called a "hundred."

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

Second grade students build on their previous work with groups of tens to make bundles of 100s, with or without leftovers, using base-ten blocks, cubes in towers of 10, ten frames, etc. and math drawings that initially show the ten tens within one hundred but then move to a quick-hundred version that is a drawn square in which students visualize ten tens. Bundling hundreds will support students' discovery of place value patterns (**MP.7**). Students explore the idea that numbers such as 100, 200, 300, etc., are groups of hundreds that have "0" in the tens and ones places. Students might represent numbers using place value (base ten) blocks (**MP.1**).

Examples: Recognizing 10 tens as 1 hundred.
Using Base-Ten Blocks:

These have the same value: 	Six hundreds is the same as 600: 
Using Math Drawings:	
When I bundle 10 “ten-sticks” I get 1 “hundred-flat.” 	The picture shows 3 hundreds, or 300. 

As students represent various numbers, they associate number names with number quantities (**MP.2**). For example, 243 can be expressed as both “2 groups of hundred, 4 groups of ten and 3 ones” and “24 tens and 3 ones.” Students can read number names as well as place value concepts to say a number. For example, 243 should be read as “two hundred forty-three” as well as “2 hundreds, 4 tens, and 3 ones.” Flexibility with seeing a number like 240 as “2 hundreds and 4 tens” as well as “24 tens” is an important indicator of place-value understanding.

In kindergarten, students were introduced to counting by tens. In second grade they extend this to skip count by **2s**, 5s, 10s and 100s **(2.NBT.2▲)**. Exploring number patterns can help students skip count. For example, when skip counting by 5s, the ones digit alternates between 5 and 0, and when skip counting by 10s and 100s, only the tens and hundreds digits change, increasing by one each time. In this way, skip counting can reinforce students’ place value understanding. Work with skip counting lays a foundation for multiplication;

however, since students do not keep track of the number of groups they have counted they are not yet learning true multiplication. The ultimate goal is for second graders to count in multiple ways without visual support.

Focus, Coherence, and Rigor:

As students explore number patterns to skip-count they also develop mathematical practices such the meaning of written quantities (**MP.2**) and number patterns and structures in the number system (**MP.7**).

Grade two students need opportunities to read and represent numerals in various ways **(2.NBT.3▲).** For example:

- Standard form (e.g., 637)
- Base-ten numerals in standard form (e.g., 6 hundreds, 3 tens and 7 ones)
- Number names in word form (e.g., six hundred thirty seven)
- Expanded form (e.g., 600 + 30 + 7)
- Equivalent representations (e.g., 500 + 130 + 7; 600 + 20 + 17; 30 + 600 + 7)

When students read the expanded form for a number, they might say "6 hundreds plus 3 tens plus 7 ones" or "600 plus 30 plus 7." Expanded form is a valuable skill when students use place value strategies to add and subtract large numbers (see also **2.NBT.7**).

Second grade students use the symbols for greater than (>), less than (<) and equal to (=) to compare numbers within 1000 **(2.NBT.4▲)**. Students build on work in standards (**2.NBT.1▲**) and (**2.NBT.3▲**) by examining the amounts of hundreds, tens, and ones in each number. To compare numbers, students apply their understanding of place value. The goal is for students to understand they look at the numerals in the hundreds place first, then the tens place, and if necessary the ones place. Students should have experience communicating their comparisons in words before using only symbols to indicate greater than, less than, and equal to.

Example: Compare 452 and 455.
Student 1: Student might explain 452 has 4 hundreds 5 tens and 2 ones and 455 has 4 hundreds 5 tens and 5 ones. They have the same number of hundreds and the same number of tens, but 455 has 5 ones and 452 only has 2 ones. So, 452 is less than 455 or 452 < 455.
Student 2: Student might think 452 is less than 455. I know this because when I count up I say 452 before I say 455.

As students compare numbers they also develop mathematical practices such as making sense of quantities (**MP.2**), understanding the meaning of symbols (**MP.6**), and making use of number patterns and structures in the number system (**MP.7**).

Number and Operations in Base Ten	2.NBT

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

7.1 Use estimation strategies to make reasonable estimates in problem solving. CA

8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

Standards **(2.NBT.5-7▲**) are crucial for attaining one of the four critical areas of instruction in grade four. It is here that students apply models of addition and subtraction to develop, discuss and later use efficient, accurate, and

[3] Explanations may be supported by drawings or objects.

generalizable methods to compute sums and differences of whole numbers in base-ten notation. While students become fluent in such methods within 100 at grade two, they also use these methods for sums and differences within 1000. The narrative will first present general written strategies for numbers within 1000, since they are simply extensions of those for numbers within 100. Of course, all methods for adding and subtracting two- and three-digit numbers should be based on place value and should be learned by students with an emphasis on understanding. Math drawings should accompany such written methods as students become familiar with them.

Written methods for addition and subtraction are based on two important features of the base-10 number system:

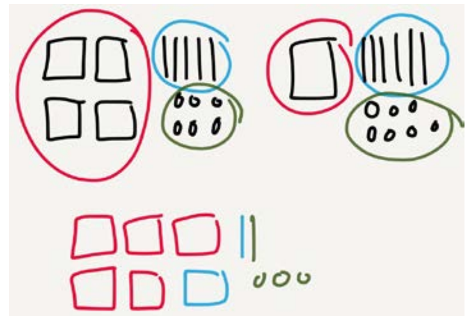
- When adding or subtracting numbers in the base-10 system, like units are added or subtracted (e.g., ones are added to ones, tens to tens, hundreds to hundreds).
- Adding and subtracting multi-digit numbers written in base-10 can be facilitated by composing and decomposing units appropriately, so as to reduce the methods to simply doing additions and subtractions within 20 (e.g., 10 ones make 1 ten, 100 ones make 1 hundred, 1 hundred makes 10 tens).

The following table illustrates two written methods for addition, with accompanying illustrations (base-10 blocks can also be used to illustrate). Students first work with math drawings or manipulatives alongside the written methods; they will eventually move on to just using written methods, mentally constructing pictures as necessary and using other strategies. Teachers should note the importance of these methods; they generalize to larger numbers and decimals and emphasize the regrouping nature of combining units. Note that these two methods are only examples and are not meant to represent all such place value methods.

The following table also illustrates two methods for subtraction, one where all decomposing is done first, the other where decomposing is done as needed. Students will encounter situations where students “don’t have enough” to subtract. Note that this is more precise than saying, “You can’t subtract a larger number from a smaller number,” or the like, as the latter statement is a false mathematical statement. Eventually, students will subtract and obtain negative numbers. Note the accompanying diagrams that show the decomposing steps in each written subtraction method. Again, these methods generalize to numbers of all sizes, and are based on decomposing larger units into smaller units when necessary.

Examples: Addition Methods Supported with Drawings.

Addition Method 1: In this written addition method, all partial sums are recorded underneath the addition bar. This particular example shows the addition being performed from left to right, but students can also do this from right to left. In the accompanying drawing, it is clear that hundreds are added to hundreds, tens to tens, and ones to ones, which are eventually grouped into larger units where possible to represent the total, 623.



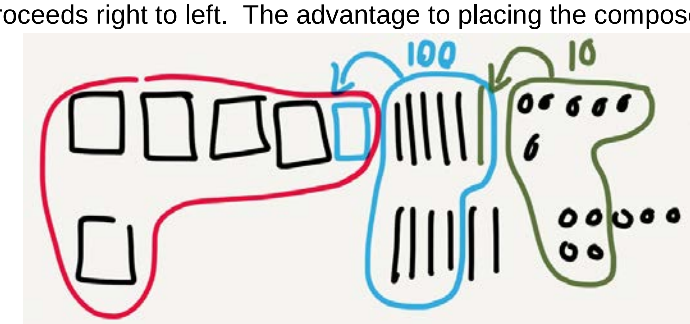
	4	5	6
+	1	6	7

	4	5	6
+	1	6	7
	5	0	0

	4	5	6
+	1	6	7
	5	0	0
	1	1	0

	4	5	6
+	1	6	7
	5	0	0
	1	1	0
		1	3
	6	2	3

Addition Method 2: In this written addition method, digits representing newly composed units are placed below the addends from which they were derived, to the right to indicate that they are represented a newly composed, larger unit. The addition proceeds right to left. The advantage to placing the composed units as shown is that it is clearer where they came from, e.g., the "1" and "3" that came from the sum of the ones-place digits (6 +7) are close to each other. This eliminates confusion that can arise from traditional methods involving "carrying", which tends to separate the two digits that came from 13 and obscure the meaning of the numbers.



	4	5	6
+	1	6	7

	4	5	6
+	1	6	7
		1	
			3

Add the ones, 6+7, and record these 13 with 3 in the ones place and a 1 underneath the tens column.

	4	5	6
+	1	6	7
	1	1	
		2	3

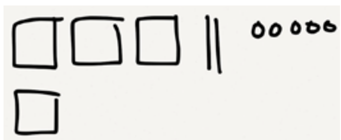
Add the tens, 5+6+1, and record these 12 tens with 2 in the tens place and 1 under the hundreds column.

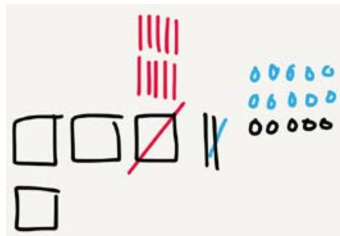
	4	5	6
+	1	6	7
	1	1	
	6	2	3

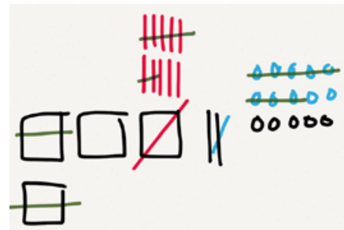
Add the hundreds, 4+1+1 and record these 6 hundreds in the hundreds column.

Examples: Subtraction Methods Supported with Drawings.

Subtraction Method 1: In this written subtraction method, all necessary decompositions are done first. Decomposing can start from the left or the right with this method. Students may be less likely to erroneously subtract the top number from the bottom in this method.







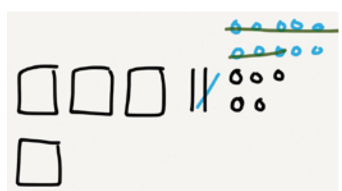
$$\begin{array}{cccc} & 4 & 2 & 5 \\ - & 2 & 7 & 8 \\ \hline \end{array}$$

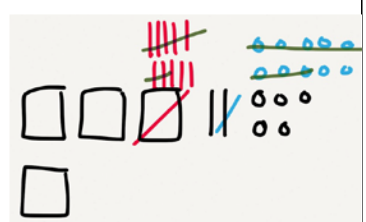
$$\begin{array}{cccc} & & 11 & \\ & 3 & \cancel{12} & 15 \\ & \cancel{4} & \cancel{2} & \cancel{5} \\ - & 2 & 7 & 8 \\ \hline \end{array}$$

decomposing left to right,
1 hundred, then 1 ten

$$\begin{array}{cccc} & & 11 & \\ & 3 & \cancel{12} & 15 \\ & \cancel{4} & \cancel{2} & \cancel{5} \\ - & 2 & 7 & 8 \\ \hline & 1 & 4 & 7 \end{array}$$

Subtraction Method 2: In this written subtraction method, decomposing is done as needed. Students first ungroup a ten so they can subtract 8 from 15. They may erroneously try to subtract the tens as well, getting 6 – 4 = 2. Led to see their error, students find they need to ungroup hundreds first to subtract the tens, then the hundreds.





$$\begin{array}{cccc} & 4 & 2 & 5 \\ - & 2 & 7 & 8 \\ \hline \end{array}$$

$$\begin{array}{cccc} & & 1 & 15 \\ & 4 & \cancel{2} & \cancel{5} \\ - & 2 & 7 & 8 \\ \hline & & & 7 \end{array}$$

$$\begin{array}{cccc} & & 11 & \\ & 3 & \cancel{1} & 15 \\ & \cancel{4} & \cancel{2} & \cancel{5} \\ - & 2 & 7 & 8 \\ \hline & 1 & 4 & 7 \end{array}$$

(Adapted from Fuson, Beckmann NCTM 2012-2013 and Progressions K-5 NBT 2011)

When developing fluency in second grade with adding and subtracting within 100 (**2.NBT.5▲**), students use the previous methods without the support of drawings, as well as various other strategies.

Strategies for Addition and Subtraction

Addition strategies based on place value for 48 + 37 may include:

- Adding by place value: 40 + 30 = 70 and 8 + 7 = 15 and 70 + 15 = 85.
- Incremental adding (by tens and ones); 48 + 10 = 58, 58 + 10 = 68, 68 + 10 = 78, 78 + 7 = 85
- Composing and decomposing (making a “friendly” number): 48 + 2 = 50, 37 – 2 = 35, 50 + 35 = 85

Subtraction strategies based on place value for 81 – 37 may include:

- Adding up (from smaller number to larger number): 37 + 3 = 40, 40 + 40 = 80, 80 + 1 = 81, and 3 + 40 + 1 = 44.
- Incremental subtracting: 81 – 10 = 71, 71 – 10 = 61, 61 – 10 = 51, 51 – 7 = 44
- Subtracting by place value: 81 – 30 = 51, 51 – 7 = 44

As students develop fluency with adding and subtracting within 100, they also support mathematical practices such as making sense of quantities (**MP.2**), calculating accurately (**MP.6**), and making use of number patterns and structures in the number system (**MP.7**).

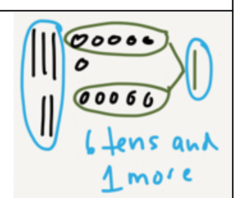
Example: Find the sum, 43 + 34 + 57 + 24.
Student A (Commutative and Associative Properties): I saw the 43 and 57 and added them first. I know 3 plus 7 equals 10, so when I added them 100 was my answer. Then I added 34 and had 134. Then I added 24 and had 158. So 43 + 57 + 34 + 24 = 158.
Student B (Place Value Strategies): I broke up all of the numbers into tens and ones. First I added the tens. 40 + 30 + 50 + 20 = 140. Then I added the ones. 3 + 4 + 7 + 4 =18. That meant I had 1 ten and 8 ones. So, 140 + 10 is 150. 150 and 8 more is 158. So, 43 + 34 + 57 + 24 = 158.
Student C (Place Value Strategies and Commutative and Associative Property): I broke up all the numbers into tens and ones. First I added up the tens, 40 + 30 + 50 + 20. I changed the order of the numbers to make adding easier. I know that 30 plus 20 equals 50 and 50 more equals 100. Then I added the 40 and got 140. Then I added up the ones. 3 + 4 + 7 + 4. I changed the order of the numbers to make adding easier. I know that 3 plus 7 equals 10 and 4 plus 4 equals 8. 10 plus 8 equals 18. I then combined my tens and my ones. 140 plus 18 (1 ten and 8 ones) equals 158.

Finally, students explain why addition and subtraction strategies work, using place value and the properties of operations. (**2.NBT.9▲**) Second grade students need multiple opportunities to explain their addition and subtraction thinking (**MP.2**). For example, students use place value understanding, properties of operations, number names, words (including mathematical language), math drawings, number lines, and/or physical objects to explain why and how they solve a problem (**MP.1, MP.6**). Students can also critique the work of other students (**MP.3**) to deepen their understanding of addition and subtraction strategies.

Example: There are 36 birds in the park. 25 more birds arrive. How many birds are there? Solve the problem and show your work.

Student A: I broke 36 and 25 into tens and ones (30 + 6) + (20 + 5). I can change the order of my numbers, since it doesn't change any amounts, so I added 30 + 20 and got 50. Then I added 5 and 5 to make 10 and added it to the 50. So, 50 and 10 more is 60. I added the one that was left over and got 61. So there are 61 birds in the park.

Student B: I used a math drawing and made a pile of 36 and a pile of 25. Altogether, I had 5 tens and 11 ones. 11 ones is the same as one ten and one left over. So, I really had 6 tens and 1 one. That makes 61.



[Note: Sidebar]

Focus, Coherence, and Rigor:

When students explain why addition and subtraction strategies work (**2.NBT.9▲**), they reinforce foundations for solving one- and two-step word problems (**2.OA.1▲**) and extend their understanding and use of various strategies and models, drawings, and a written method to add and subtract (**2.NBT. 5▲ and 7▲**).

Students are to fluently add and subtract within 100 in grade two (**2.NBT. 5▲**). Students added within 100 in grade one using concrete models or drawings and used at least one method that is generalizable to larger numbers (such as between 101 and 1000). Students extend to larger numbers using drawings and written methods as discussed previously. This extension could be connected first to adding all 2-digit numbers (e.g., 78 + 47) so that students can see and discuss composing both ones and tens without the complexity of hundreds in the drawings or numbers (refer to the earlier 3-digit addition examples on page 20, without the hundreds in the problems or in the drawings). After solving additions that compose both ones and tens for all 2-digit numbers and then within 1000, the fluency problems for grade two seem easy: 28 + 47 requires only composing the ones. This is now easier to do without drawings: one just records the new ten before it is added in or adds it in mentally. Fluent adding means adding without drawings.

The same approach can be taken for subtraction, first briefly solving with concrete models or drawings of subtractions within 100 that involve ungrouping one ten to make ten ones and then solving subtractions that require two decompositions, of one hundred to make ten tens and of one ten to make ten ones. Spending a long time subtracting within 100 initially can stimulate students to count/on or count down, methods that become considerably more difficult above 100. Problems with all possibilities of decompositions should be mixed in so that students solve problems requiring two, one, and no decompositions. Then students can spend time on subtractions that include multiple hundreds (totals from 201 to 1000). After this experience, focusing within 100 just on the two cases of one decomposition (e.g., 73 – 28) or no decomposition (e.g., 78 – 23) is relative easy to do without drawings.

“Mental Math” **as an instructional tool.** Many teachers incorporate a powerful activity known as “Mental Math” in their classrooms. The teacher typically writes an appropriate problem on the board (such as 45 + 47) and asks students to solve the problem mentally only. The teacher then records all answers given by students, regardless of being correct or incorrect, without judgment. A class discussion follows with students explaining how they got their answers, and the students decide which answer is correct. The class may agree or disagree with a particular method, find out where another student made an error, or compare different solution methods (e.g. how finding 45 + 45 + 2 is similar to finding 40 + 40 + 12). In Mental Math, often multiple strategies emerge naturally from the students themselves, and opportunities to explore these strategies arise.

When students do not have more than one strategy for solving a problem, this can be an indication to the teacher that students have a limited repertoire of such strategies, and therefore Mental Math can be used as a formative instruction tool. Mental Math supports several Mathematical Practice standards, including **MP.1, MP.2, MP.3, MP.7, and MP.8.**

Domain: Measurement and Data

Grade one students worked with linear measurement using nonstandard units. A critical area of instruction in grade two is for students to use standard units of measure.

Measurement and Data	2.MD

Measure and estimate lengths in standard units.

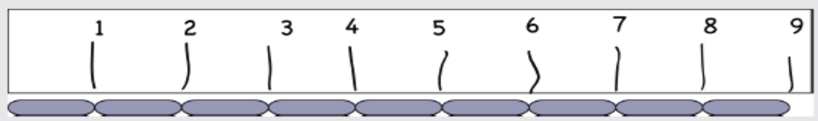
1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3. Estimate lengths using units of inches, feet, centimeters, and meters.
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Second graders are transitioning from measuring lengths with informal or nonstandard units to measuring with standard units—inches, feet, centimeters, and meters—and using standard measurement tools **(2.MD.1▲)**. Students learn the measure of length as a count of how many units are needed to match the length of the object or distance being measured. Using both customary (inches and feet) and metric (centimeters and meters) units, students measure the length of objects with rulers, yardsticks, meter sticks, and tape measures. Students become familiar with standard units (e.g., 12 inches in a foot, 3 feet in a yard, and 100 centimeters in a meter) and how to estimate lengths. (Adapted from KATM 2nd FlipBook 2012).

Students also can learn accurate procedures and concepts by drawing simple unit rulers. Using copies of a single length-unit such as inch-long manipulatives, students mark off length-units on strips of paper, explicitly connecting measurement with the ruler to measurement by iterating physical units.

Example of a student created ruler:

Using a unit to draw a ruler



Students use a standard unit (shown in below the ruler) to make rulers, helping them to understand the meaning of the markings on rulers.

(Adapted from Progressions K-5 MD, measurement part 2012)

Thus, students’ first rulers are simple tools to help count the iteration of length-units. Frequently comparing results of measuring the same object with manipulative standard units and with student-created rulers can help students connect their experiences and ideas. As they build and use these tools, they develop the ideas of length-unit iteration, correct alignment (with a ruler), and the zero-point concept (the idea that the zero of the ruler indicates one endpoint of a length).

Second graders learn the concept of the inverse relationship between the size of the unit of length and the number of units required to cover a specific length or distance, specifically, that the larger the unit, the fewer units needed to measure something, and vice versa **(2.MD.2▲)**.

Second graders learn the concept of the inverse relationship between the size of the unit of length and the number of units required to cover a specific length or distance.

Students measure the length of the same object using units of different lengths (ruler with inches vs. ruler with centimeters or a foot ruler vs. a yardstick) and discuss the relationship between the size of the units and measurements.

Example: A student measured the length of a desk in both feet and inches. The student found that the desk was 3 feet long and that it was 36 inches long.
Teacher: "Why do you think you have two different measurements for the same desk?" **Student:** "It only took 3 feet because the feet are so big. It took 36 inches because an inch is much smaller than a foot."

Students use this information to understand how to select appropriate tools for measuring a given object. For instance, a student might think, "The longer the unit, the fewer I need." Measurement problems also support mathematical practices such reasoning quantitatively (**MP.2**), justifying conclusions (**MP.3**), using appropriate tools (**MP.5**), attending to precision (**MP.6**), and making use of structure or patterns (**MP. 7**).

Students estimate lengths using units of inches, feet, centimeters, and meters. (**2.MD.A.3▲**). Students estimate lengths before they measure. After measuring an object, students discuss their estimations, measurement procedures, and the differences between their estimates and the measurements. Students should begin by estimating measurements of familiar items (length of desk, pencil, favorite book, etc.). Estimation helps students focus on the attribute to be measured, the length units, and the process. Students need many experiences with using measuring tools to develop their understanding of linear measurement. For example:

Teacher: "How many inches do you think this string is if you measured it with a ruler?"

Student: "An inch is pretty small. I'm thinking it will be somewhere between 8 and 9 inches."
Teacher: "Measure it and see."
Student: "It is 9 inches. I thought that it would be somewhere around there."

The previous example also supports mathematical practices such making sense of quantities (**MP.2**) and appropriate use of tools (**MP.5**).

Students measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (**2.MD.A.4▲)**.

Second graders use inches, feet, yards, centimeters, and meters to compare the lengths of two objects. Students use comparative phrases such as, "It is 2 inches longer," or, "It is shorter by 5 centimeters," to describe the difference in length between the two objects. Students use both the quantity and the unit name to precisely compare length.
(Adapted from Arizona 2012 and N. Carolina 2013)

Focus, Coherence, and Rigor. As student compare objects by their lengths they also reinforce skills and understanding related to solving compare problems in the major cluster "Represent and solve problems involving addition and subtraction." Drawing comparison bars to represent their measure situation helps make this link explicit. (See standard **2.OA.1▲**)

Measurement and Data 2.MD

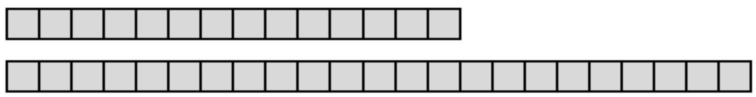
Relate addition and subtraction to length.

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Students apply the concept of length to solve addition and subtraction problems. Word problems should refer to the same unit of measure (**2.MD.5▲)**.

Example: In gym class Kate jumped 14 inches. Lilly jumped 23 inches. How much farther did Lilly jump than Kate? Solve the problem and then write an equation.

Student A: My equation is 14 + __ = 23 since I thought, “14 and what makes 23?” I used cubes. I made a train of 14. Then I made a train of 23. When I put them side by side, I saw that Kate would need 9 more cubes to be the same as Lilly. So, Lilly jumped 9 more inches than Kate. 14 + 9 = 23. (**MP.1, MP.2 and MP.4**)



Student B: My equation is 23 – 14 = __ since I thought about what the difference was between Kate and Lilly. I broke up 14 into 10 and 4. I know that 23 minus 10 is 13. Then, I broke up the 4 into 3 and 1. 13 minus 3 is 10. Then, I took one more away. That left me with 9. So, Lilly jumped 9 inches more than Kate. That seems to make sense since 23 is almost 10 more than 14. 23 – 14 = 9. **(MP.2, MP.7 and MP.8)**

Focus, Coherence, and Rigor:

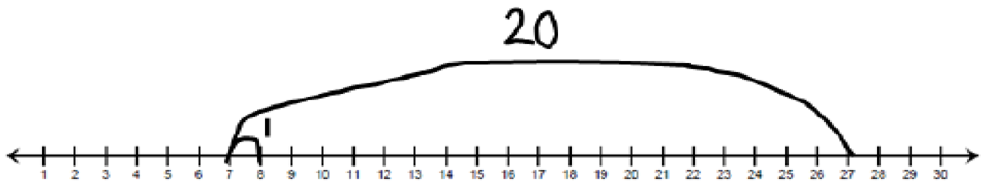
Addition and subtraction word problems involving lengths develop mathematical practices such making sense of problems (**MP.1**), reasoning quantitatively (**MP.2**), justifying conclusions (**MP.3**), appropriate use of tools (**MP.5**), attention to precision (**MP.6**), and evaluating the reasonableness of results (**MP. 8**). Similar word problems also support students’ ability to fluently add and subtract, which is part of the major work at the grade (refer to fluency expectations in standards **2.OA. 1▲** and **2.NBT. 5▲**).

Students represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2...and represent whole-number sums and differences within 100 on a number line diagram. (**2.MD. 6▲**)

Second grade students can create number lines with evenly spaced points. They discuss the similarities between a number line and a ruler. Students use a number line to add and subtract within 100 and demonstrate their thinking.

Example: There were 27 students on the bus. 19 got off the bus. How many students are on the bus?

Student: I used a number line. I saw that 19 is really close to 20. Since 20 is a lot easier to work with, I took a jump of 20. But, that was one too many. So, I took a jump of 1 to make up for the extra. I landed on 8. So, there are 8 students on the bus. So what I did was 27 – 20 = 7 and then



7 + 1 = 8

[Note: Sidebar]

Focus, Coherence, and Rigor:

Using addition and subtraction within 100 to solve word problems involving length **(2.MD.5**) and representing sums and differences on a number line **(2.MD.6**) reinforces the use of models to add and subtract and supports major work at the grade (see standards **2.OA.A.1▲** and **2.NBT.7▲**). Similar problems also develop mathematical practices such making sense of problems (**MP.2**), justifying conclusions (**MP.3**), and modeling mathematics (**MP.4**).

(Adapted from Arizona 2012 and N. Carolina 2013)

Measurement and Data 2.MD

Work with time and money.

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **Know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year). CA**
8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

In first grade, students learned to tell time to the nearest hour and half-hour. In second grade students tell time to the nearest five minutes (**2.MD.7▲)**. Students can make connections between skip counting by 5s (**2.NBT.2▲**) and 5-minute intervals on the clock. Students work with both digital and analog clocks. They recognize time in both formats and communicate their understanding of time using both numbers and language.

Second grade students understand that there are two cycles of twelve hours in a day—a.m. and p.m. A daily journal can help students make real-world connections and understand the difference between these two cycles.

[Note; Sidebar]

Focus, Coherence, and Rigor:
Students understanding and use of skip counting by 5s and 10s (**2.NBT.2▲**) can also support telling and writing time to the nearest five minutes (**2.MD. ▲7**). Students notice the pattern of numbers and apply this understanding to time (**MP.7**)

Students solve word problems involving dollars or cents **(2.MD.8)**. Students identify, count, recognize, and use coins and bills in and out of context. They should have opportunities to make equivalent amounts using both coins and bills. "Dollar bills" should include denominations up to one hundred ($1, $5, $10, $20, $100). Note that students in second grade *do not* express money amounts using decimal points.

Just as students learn that a number (38) can be represented different ways (3 tens and 8 ones; 2 tens and 18 ones) and still remain the same amount (38), students can apply this understanding to money. For example, 25 cents could be represented as a quarter, two dimes and a nickel, or 25 pennies, all of which have the same value. Building the concept of equivalent worth takes time and students will need numerous opportunities to create and count different sets of coins and to recognize the "purchase power" of coins (a nickel can buy the same things as 5 pennies).

As teachers provide students with opportunities to explore coin values (25 cents), actual coins (2 dimes, 1 nickel), and drawings of circles that have values indicated, students gradually learn to mentally give each coin in a set a value, place a random set of coins in order, use mental math, add on to find differences, and skip count to determine the total amount.

Example: How many different ways can you make 37¢ using pennies, nickels, dimes, and quarters?
Example: How many different ways can you make 12 dollars using $1, $5, and $10 bills?

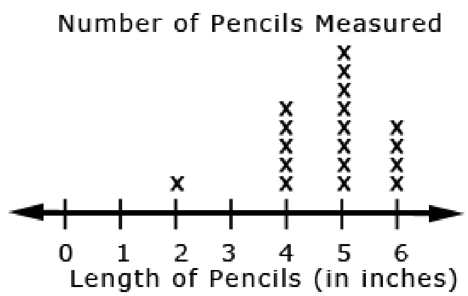
(Adapted from Arizona 2012 and N. Carolina 2013)

Measurement and Data 2.MD

Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph.

Students use the measurement skills learned in earlier standards to measure objects and create measurement data (**2.MD.9)**. For example they measure objects in their desk to the nearest inch, display the data collected on a line plot, and answer related questions. Line plots are first introduced in this grade level. A line plot can be thought of as plotting data on a number line. For example:



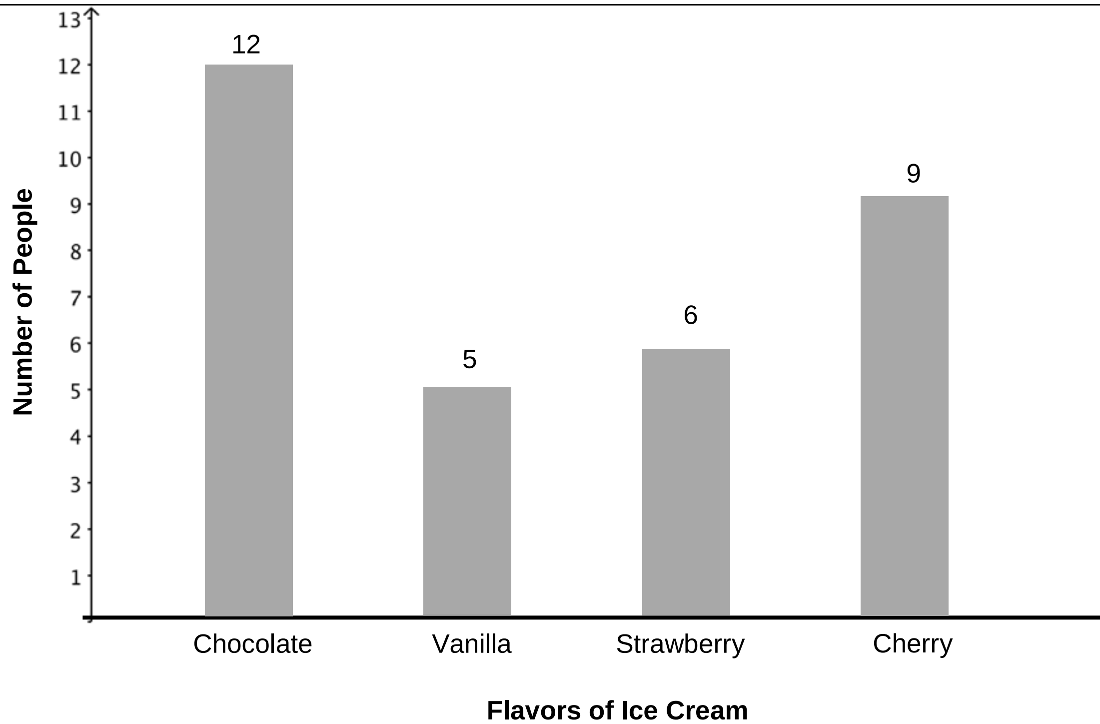
Students draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. They solve simple put-together, take-apart, and compare problems using information presented in a bar graph. (**2.MD.D.10**)

[4] See Glossary, Table 1.

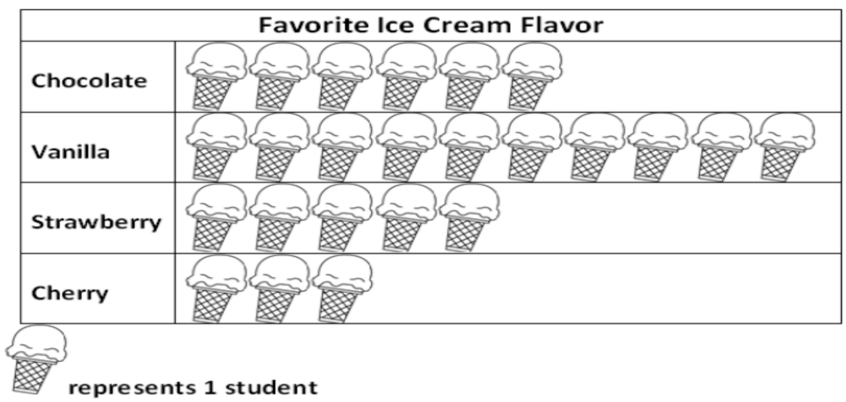
In first grade, students worked with three categories of data. In second grade, students represent data on a picture graph or bar graph (with single-unit scale) and interpret the results. Students organize, represent, and interpret data with up to four categories. In second grade, picture graphs (pictographs) use symbols that represent single units. Pictographs should include a title, categories, category label, key, and data.

Students use data to pose and solve simple one-step addition and subtraction problems. The use of picture graphs and bar graphs to represent a data set (**2.MD.D.10**) reinforces major work at the grade in the cluster “Represent and solve problems involving addition and subtraction” and provides a context for students to solve related addition and subtraction problems (**2.OA.A.1▲**).

Example: Students are responsible for purchasing ice cream for an event at school. They decide to collect data to determine which flavors to buy for the event. Students decide on the question, “What is your favorite flavor of ice cream?” and four likely responses, chocolate, vanilla, strawberry, and cherry. Students form two teams and collect information from different classes in their school. Each team decides how to keep track of the data (e.g., tally marks, in a table, check marks). Each team selects either a picture graph or a bar graph to display their data. They create the graph using paper or a computer. Examples of graphs are provided below.
Team A: Bar Graph



Team B: Picture Graph



The teacher facilitates a discussion around the data collected, asking questions like: “Based on the graph from Team A, how many students voted for cherry, strawberry, vanilla or chocolate ice cream? Based on the graph from Team B, how many students voted for cherry, strawberry, vanilla or chocolate ice cream? Using the data from both teams, which flavor got the most votes, the fewest votes? Which flavor was the second favorite flavor? Based on the data what flavors of ice cream do you think we should purchase for our event and why?”

Representing and interpreting data to solve problems also develops mathematical practices such making sense of problems (**MP.1**), reasoning quantitatively (**MP.2**), justifying conclusions (**MP.3**), appropriate use of tools (**MP.5**), attention to precision (**MP.6**), and evaluating the reasonableness of results (**MP. 8**).

Domain: Geometry

Grade one students reasoned about attributes of geometric shapes. A critical area of instruction in second grade is for students to describe and analyze shapes by examining their sides and angles. This work will develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Geometry	2.G

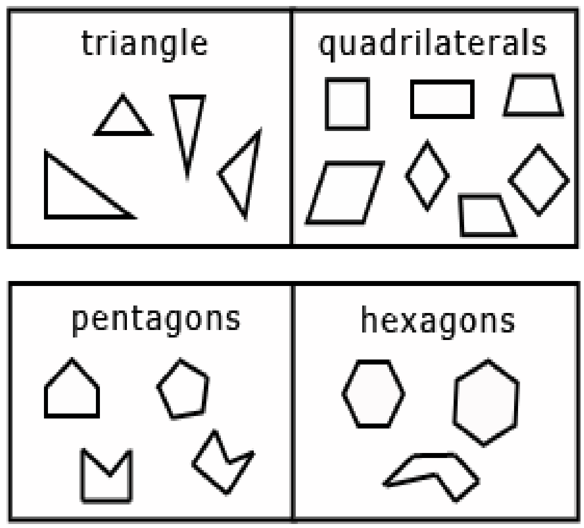
Reason with shapes and their attributes.

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Students identify, describe, and draw triangles, quadrilaterals (squares, rectangles and parallelograms, and trapezoids), pentagons, hexagons, and cubes **(2.G.1)**. Pentagons, triangles, and hexagons should appear as both regular (equal sides and equal angles) and irregular. Students recognize all four sided shapes as quadrilaterals. Students use the vocabulary word “angle” in place of “corner,” but they do not need to name angle types (e.g. right, acute,

[5] Sizes are compared directly or visually, not compared by measuring.

obtuse). Shapes should be presented in a variety of orientations and configurations.

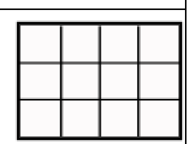


As students use attributes to identify and describe shapes they also develop mathematical practices such as analyzing givens and constraints (**MP.1**), justifying conclusions (**MP.3**), modeling with mathematics (**MP.4**) appropriate use of tools (**MP.5**), attention to precision (**MP.6**), and looking for a pattern or structure (**MP. 7**).

Students partition a rectangle into rows and columns of same-size squares and count to find the total number of squares. (**2.G.2**) As students partition rectangles into rows and columns they build a foundation for learning about the area of a rectangle and using arrays for multiplication.

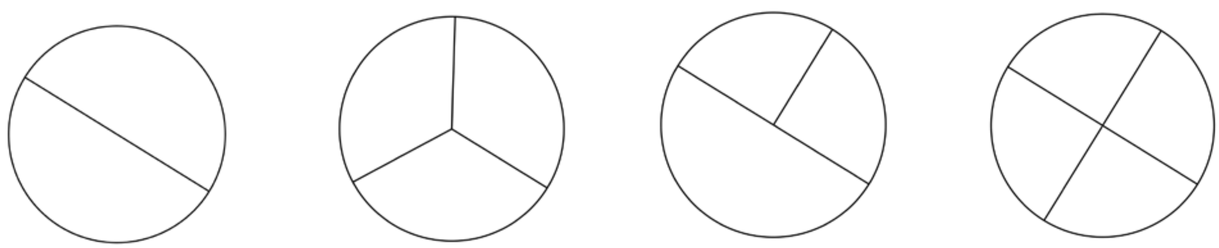
Example: Partition the rectangle into 3 equal rows and 4 equal columns. How can you partition into 3 equal rows? Then into 4 equal columns? Can you do it in the other order? How many small squares did you make?

Student: "I counted 12 squares in this rectangle. This is a lot like when we counted arrays by counting 4+4+4=12."



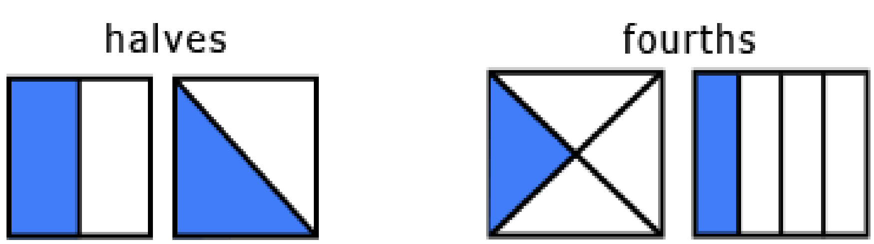
An interactive whiteboard or manipulatives such as square tiles, cubes, or other square-shaped objects can be used to help students partition rectangles (**MP.5**).

In first grade students partitioned shapes into halves, fourth and quarters. Second grade students partition circles and rectangles into 2, 3 or 4 equal shares (regions). Students explore this concept with paper strips and pictorial representations and work with the vocabulary terms halves, thirds, halves, and fourths. (**2.G.3**) Students recognize that when they cut a circle into three equal pieces, each piece will equal one third of its original whole and students describe the whole as three thirds. If a circle is cut into four equal pieces, each piece will equal one fourth of its original whole and the whole is described as four fourths.



Circle cut into halves | Circle cut into thirds | Circle **NOT** cut into thirds | Circle cut into fourths

Students should see circles and rectangles partitioned in multiple ways so they learn to recognize that equal shares can be different shapes within the same whole.



As students partition circles and squares and explain their thinking they develop mathematical practices such as making sense of quantities (**MP.2**), justifying conclusions (**MP.3**), attending to precision (**MP.6**), and evaluating the reasonableness of their results (**MP. 7**). They also develop understandings that will support major work at grade three in the cluster “Develop understanding of fractions as numbers”.

(Adapted from Arizona 2012 and N. Carolina 2013)

Essential Learning for the Next Grade

In kindergarten through grade five, the focus is on the addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals, with a balance of concepts, skills and problem solving. Arithmetic is viewed as an important set of skills and also as a thinking subject that, done thoughtfully, prepares students for algebra. Measurement and geometry develop alongside number and operations and are tied specifically to arithmetic along the way.

In kindergarten through grade two students focus on addition and subtraction and measurement using whole numbers. To be prepared for grade three mathematics, students should be able to demonstrate they have acquired certain mathematical concepts and procedural skills by the end of grade two and have met the fluency expectations for the grade. For second graders, the expected fluencies are add and subtract within 20 using mental strategies and know from memory all sums of two one-digit numbers (**2.OA.2▲**), and add and subtract within 100 using various strategies (**2.NBT.5▲)**. These fluencies and the conceptual understandings that support them are foundational for work in later grades.

Of particular importance at grade two are concepts, skills, and understandings of addition and subtraction within 20 and representing and solving problems involving addition and subtraction (**2.OA.1-2▲)**; place value (**2.NBT1-4▲**) and the use of place value understanding and properties of operations to add and subtract (**2.NBT.5-9▲**); measuring and estimating lengths in standard units (**2.MD.1-4▲**) and relating addition and subtract to length. (**2.MD.5-6▲)**

Place Value

By the end of grade two students are expected to read, write and count to 1000; skip-counting by 2s, 5s, 10s and 100s. Students need to understand 100 can be thought of as a bundle of ten tens and also understand three-digit whole numbers in terms of hundreds, tens and ones.

Addition and Subtraction

Addition and subtraction are major instructional focuses in kindergarten through grade two. By the end of grade two students are expected to add and subtract (using concrete models, drawings and strategies) within 1000 (**2.NBT.7▲**). Students should add and subtract fluently within 100, using various strategies, (**2.NBT.5▲**) and fluently within 20, using mental strategies (**2.OA.2▲**). Students mentally add and subtract 10 or 100, within the range 100-900 (**2.NBT.8▲**). Students are expected to know from memory all sums of two one-digit numbers (**2.OA.2▲**). Students should know how to apply addition and subtraction to solve a variety of one- and two-step word problems (within 100) involving add-to, take-from, put-together, take-apart, and compare situations (**2.OA.1▲**). Refer to the table on page 7 for additional information.

Students who have met the grade two standards for addition and subtraction will be prepared to fluently add and subtract within 1000 using strategies and algorithms as required in the grade three standards. These foundations will also prepare students for concepts, skills, and problem solving with multiplication and division, which are introduced in grade three.

Measurement

By the end of grade two, students can measure lengths using standard—units—inches, feet, centimeters, and meters. Students need to know how to use addition and subtraction within 100 to solve word problems involving lengths (**2.MD.5▲**). Mastering grade two measurement standards will prepare students to measure fractional amounts and to add, subtract, multiply, or divide to solve word problems involving mass or volume as required in the grade three standards.

Grade 2 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 2

Operations and Algebraic Thinking 2.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten 2.NBT

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
    a. 100 can be thought of as a bundle of ten tens — called a “hundred.”
    b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by **2s**, 5s, 10s, and 100s. **CA**
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

7.1 Use estimation strategies to make reasonable estimates in problem solving. CA

8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

[1]See Glossary, Table 1.
[2]See standard 1.OA.6 for a list of mental strategies.

[3]Explanations may be supported by drawings or objects.

Measurement and Data 2.MD

Measure and estimate lengths in standard units.

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3. Estimate lengths using units of inches, feet, centimeters, and meters.
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. ***Know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year).*** **CA**
8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
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[4]See Glossary, Table 1.
[5]Sizes are compared directly or visually, not compared by measuring.