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Glossary: Mathematical Terms, Tables, and Illustrations

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25 Adapted from the *Massachusetts Curriculum Framework for Mathematics*, March 2011.

26 This glossary contains those terms found and defined in the Common Core State

27 Standards Initiative's glossary for the Common Core State Standards for Mathematics,

28 as well as many additional terms.

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31 **AA similarity.** Angle-angle similarity. When two angles of one triangle are congruent to

32 two angles of a second triangle, the triangles are similar.

33 **ASA congruence.** Angle-side-angle congruence. When two triangles have

34 corresponding angles and the included side that are congruent, the triangles

35 themselves are congruent. (MW)

36 **Absolute value.** The absolute value of a number x is the nonnegative number that

37 represents its distance from 0 on a number line. Equivalently, $|x| = x$ if $x \geq 0$,

38 or $-x$ if $x < 0$."

39 **Addition and subtraction within 5, 10, 20, 100, or 1000**. Addition or subtraction of two

40 whole numbers with whole number answers, and with sum or minuend in the range 0–5,

41 0–10, 0–20, or 0–100, respectively. *Example: 8 + 2 = 10 is an addition within 10,*

42 *14 – 5 = 9 is a subtraction within 20, and   55 – 18 = 37 is a subtraction within 100.*

43 **Additive inverses.** Two numbers whose sum is 0 are additive inverses of one another.

44 Example: $\dfrac{3}{4}$ and $-\dfrac{3}{4}$ are additive inverses of one another because

45 $\dfrac{3}{4} + (-\dfrac{3}{4}) = (-\dfrac{3}{4}) + \dfrac{3}{4} = 0.$ (CCSSM)

46 **Algorithm.** A set of pre-defined steps applicable to a class of problems that give the

47 correct result in every case when the steps are carried out correctly.

48 **Analog.** Having to do with data represented by continuous variables, e.g., a clock with

49 hour, minute, and second hands. (M)

50 **Analytic geometry.** The branch of mathematics that uses functions and relations to

51 study geometric phenomena, e.g., the description of ellipses and other conic sections in

52 the coordinate plane by quadratic equations.

53 **Argument of a complex number.** The angle θ when a complex number is represented

54 in polar form $r(\cos \theta + i \sin \theta)$

55 **Associative property of addition**. See Table 3 in this Glossary.

56 **Associative property of multiplication.** See Table 3 in this Glossary.

57 **Assumption.** A fact or statement (as a proposition, axiom, postulate, or notion)

58 accepted as true.

59 **Attribute.** A common feature of a set of figures.

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60 **Benchmark fraction**. A common fraction against which other fractions can be

61 measured, such as $\frac{1}{2}$.

62 **Binomial Theorem.** The theorem that gives the polynomial expansion of each whole-

63 number power of a binomial.

64 **Bivariate data.** Pairs of linked numerical observations. *Example: a list of heights and*

65 *weights for each player on a football team.*

66 **Box plot**. A graphic that shows the distribution of data values by using the median,

67 quartiles, and extremes of the data set. A box shows the middle 50% of the data. (DPI)

68 **Calculus.** The mathematics of change and motion. The main concepts of calculus are

69 limits, instantaneous rates of change, and areas enclosed by curves.

70 **Capacity.** The maximum amount or number that can be contained or accommodated,

71 e.g., a jug with a one-gallon *capacity;* the auditorium was filled to *capacity.*

72 **Cardinal number.** A number (such as 1, 5, 15) that indicates how many elements there

73 are in a set.

74 **Cartesian plane.** A coordinate plane with perpendicular coordinate axes.

75 **Causation.** The act of causing or inducing. If one action causes another, then the

76 actions are certainly correlated. But just because two things occur together does not

77 mean that one caused the other. (S) See also: **correlation**.

78 **Cavalieri's Principle**. If, in two solids of equal altitude, the sections made by planes

79 parallel to and at the same distance from their respective bases are always equal, the

80 volumes of the two solids are equal. (K)

81 **Coefficient.** The numerical factor in a product. For example, 3 is the coefficient of *ab* in

82 the term 3*ab*.

83 **Commutative property**. See Table 3 in this Glossary.

84 **Complex fraction**. A fraction $\frac{A}{B}$ where *A* and/or *B* are fractions (*B* nonzero).

85 **Complex number**. A number that can be written in the form *a* + *bi* where *a* and *b* are

86 real numbers and i^2 = -1.

87 **Complex plane.** A Cartesian plane in which the point (*a, b*) represents the complex

88 number *a* + *bi*.

89   **Compose numbers. (V)** Given pairs, triples, etc. of numbers, to identify sums or

90   products that can be computed. For example, each place in the base ten place value is

91   composed of ten units of the place to the left, i.e., one hundred is composed of ten

92   bundles of ten, one ten is composed of ten ones, etc.

93   **Compose shapes. (V)** To join geometric shapes without overlaps to form other shapes.

94   **Composite number.** A whole number that has more than two distinct positive factors.

95   (H)

96   **Computation algorithm**. A set of predefined steps applicable to a class of problems

97   that gives the correct result in every case when the steps are carried out correctly. *See*

98   *also: **algorithm**; **computation strategy***.

99   **Computation strategy**. Purposeful manipulations that may be chosen for specific

100 problems, may not have a fixed order, and may be aimed at converting one problem

101 into another. *See also: **computation algorithm**.*

102 **Congruent**. Two plane or solid figures are congruent if one can be obtained from the

103 other by a rigid motion (a sequence of rotations, reflections, and translations).

104 **Conjugate.** The result of writing a sum of two terms as a difference, or vice versa. *For*

105 *example, the conjugate of $x - 2$ is $x + 2$.* (MW)

106 **Constant of proportionality.** The constant k in the equation $y = kx$ that shows that y is

107 directly proportional to x. The unit rate associated with a ratio is an example of a

108 constant of proportionality. *See also* **proportional relationship**.

109 **Coordinate plane.** A plane in which points are designated using two coordinates. In the

110 Cartesian or rectangular coordinate plane, the two coordinates correspond to numbers

111 on two perpendicular numbers lines, called axes, that intersect at the zero of each axis.

112 **Correlation**. A measure of the amount of positive or negative relationship existing

113 between two measures. For example, if the height and weight of a set of individuals

114 were measured, it could be said that there is a positive correlation between height and

115 weight if the data showed that larger weights tended to be paired with larger heights and

116 smaller weights tended to be paired with smaller heights. The stronger those

117 tendencies, the closer the measure is to -1 or 1. (DPI) *See also*: **causation.**

118 **Cosine.** A trigonometric function that for an acute angle of a right triangle is the ratio

119 between a leg adjacent to the angle and the hypotenuse.

120 **Counting number.** A number used in counting objects, i.e., a number from the

121 set 1, 2, 3, 4, 5,…. See Illustration 1 in this Glossary.

122 **Counting on**. A strategy for finding the number of objects in a group without having to

123 count every member of the group. *For example, if a stack of books is known to have 8*

124 *books and 3 more books are added to the top, it is not necessary to count the stack all*

125 *over again; one can find the total by counting on—pointing to the top book and saying*

126 *"eight," following this with "nine, ten, eleven. There are eleven books now."*

127 **Decimal expansion.** The representation of a real number using base-10 notation (e.g.,

128 the decimal expansion of the number ¼ is 0.25).

129 **Decimal fraction**. A fraction (such as $0.25 = \frac{25}{100}$ or $0.025 = \frac{25}{1000}$) or mixed number (as

130 $3.025 = 3\frac{25}{1000}$) in which the denominator is a power of ten. Decimal fractions are

131 usually expressed in base-10 notation with the use of the decimal point.

132 **Decimal number.** Any real number expressed in base-10 notation, such as 2.673.

133 **Decompose numbers.** To write a given number as a sum or difference of two or more

134 other numbers.

135 **Decompose shapes.** Given a geometric shape, to identify geometric shapes that meet

136 without overlap to form the given shape.

137 **Digit.** a) Any of the Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9; b) One of the elements

138 that combine to form numbers in a system other than the decimal system.

139 **Digital.** Having to do with data that is represented in the form of numerical digits;

140 providing a readout in numerical digits, e.g., a digital watch.

141 **Dilation**. A transformation that moves each point along the ray through the point

142 emanating from a fixed center and multiplies distances from the center by a common

143 scale factor. (CCSSM) A transformation in which a figure is made proportionally larger

144 or smaller.

145 **Directrix.** A straight line the distance to which from any point of a conic section is in

146 fixed ratio to the distance from the same point to the conic's focus.

147 **Discrete mathematics.** The branch of mathematics that includes combinatorics,

148 recursion, Boolean algebra, set theory, and graph theory.

149   **Dot plot.** *See: line plot.*

150   **Double Number Line Diagram.** A diagram in which two number lines subdivided in the

151   same way are set one on top of the other with zeros lined up. While the number lines

152   are subdivided in the same way, the units in each may be different, which allows for

153   illustrating ratio relationships. Double number lines can also be constructed vertically.

154   **Expanded form**. A multi-digit number is expressed in expanded form when it is written

155   as a sum of single-digit multiples of powers of ten. *For example, 643 = 600 + 40 + 3.*

156   **Expected value.** For a random variable, the weighted average of its possible values,

157   with weights given by their respective probabilities.

158   **Exponent.** For positive integer values, the number that indicates how many times the

159   base is used as a factor, e.g., in 4^3=4x4x4= 64, the exponent is 3, indicating that 4 is

160   repeated as a factor three times.

161   **Exponential function.** An exponential function is a function of the form $y(x) = a \cdot b^x$,

162   where $a \neq 0$ and either $0 < b < 1$ or $1 < b$. The variables do not have to be *x* and *y*. For

163   example $A = 3.2 \cdot (1.02)^t$ defines *A* as an exponential function of *t*.

164   **Expression.** A mathematical phrase that combines operations, numbers, and/or

165   variables (e.g.,$3^2 \div a$). (H)

166   **Extreme Values of a Polynomial**. The graph of a polynomial of degree *n* has at most *n*

167   – 1 extreme values (local minima and/or maxima). The total number of extreme values

168   could be *n* – 1 or *n* – 3 or *n* – 5 etc. For example, a degree 9 polynomial could have 8,

169   6, 4, 2, or 0 extreme values. A degree 2 (quadratic) polynomial must have 1 extreme

170   value. (MW)

171   **Fibonacci sequence.** The sequence of numbers beginning with 1, 1, in which each

172   number that follows is the sum of the previous two numbers, i.e., 1, 1, 2, 3, 5, 8, 13, 21,

173   34, 55, 89, 144….

174   **First quartile**. For a data set with median *M*, the first quartile is the median of the data

175   values less than *M*. *Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the*

176   *first quartile is 6.*[1] *See also:* **median, third quartile, interquartile range**.

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[1] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

177 **Fluency.**

178   **Conceptual fluency.** Being able to use the relevant ideas or procedures in a

179   wide range of contexts. (Smarter Balanced)

180   **Contextual fluency.** The ability to use certain facts and procedures with enough

181   facility that using them does not slow down or derail the problem solver as he or

182   she works on more complex problems. (Smarter Balanced)

183   **Procedural fluency**. Skill in carrying out procedures flexibly, accurately,

184   efficiently, and appropriately. (CCSSM)

185   **Fluency as a special case of assessing individual content standards**. Fluent

186   in the standards means fast and accurate. The word fluency was used judiciously

187   in the standards to mark the endpoints of progressions of learning that begin with

188   solid underpinnings and then pass upward through stages of growing maturity.

189   Assessing the full range of the standards means assessing fluency where it is

190   called for in the standards. (Smarter Balanced)

191 **Focus (pl. foci).**

192 One of the fixed points from which the distances to any point of a conic curve, such as

193 an ellipse or parabola, are connected by a linear relation. (O)

194 **Fraction**. A number expressible in the form $\dfrac{a}{b}$ where a is a whole number and b is a

195 positive whole number. (The word *fraction* in these standards always refers to a

196 nonnegative number.) *See also: **rational number**.*

197 **Function. (a)** A mathematical relation for which each element of the domain

198 corresponds to exactly one element of the range. **(b)** A rule that assigns to every

199 element of one set (the domain) exactly one element from another set (the co-domain).

200 **Function notation.** A notation that describes a function. For a function f, when x is a

201 member of the domain, the symbol $f(x)$ denotes the corresponding member of the

202 range (e.g., $f(x) = x + 3$).

203 **Fundamental Theorem of Algebra.** The theorem that establishes that, using complex

204 numbers, all polynomials can be factored into a product of linear terms. An alternate

205 form of the theorem asserts that any polynomial of degree n has (counting multiplicity)

206 exactly n complex roots.

207 **Geometric sequence (progression).** An ordered list of numbers that has a common

208 ratio between consecutive terms, e.g., 2, 6, 18, 54…. (H)

209 **Histogram.** A type of bar graph used to display the distribution of measurement data

210 across a continuous range.

211 **Identity property of 0**. See Table 3 in this Glossary.

212 **Imaginary number.** A complex number of the form *bi*.

213 **Independently combined probability models**. Two probability models are said to be

214 combined independently if the probability of each ordered pair in the combined model

215 equals the product of the original probabilities of the two individual outcomes in the

216 ordered pair.

217 **Initial value (of a function). (a)** For a function *f* with domain the interval [*a*,*b*], the *initial*

218 *value* of *f* is the value *f*(*a*). If the domain of *f* is discrete, then the initial value of *f* is *f*(*n*),

219 where *n* is the smallest value in the domain of *f* (should such a smallest value exist).

220 **(b)** For a function *f* with domain all real numbers, the initial value is also taken to mean

221 *f*(0).

222 **Integers (Set of)**. The set of numbers that includes whole numbers and their opposites,

223 {…-3, -2, -1, 0, 1, 2, 3…}.

224 **Interquartile range**. A measure of variation in a set of numerical data, the interquartile

225 range is the distance between the first and third quartiles of the data set. Example: For

226 the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9.

227 *See also: **first quartile, third quartile.***

228 **Inverse.** See additive inverses and multiplicative inverses.

229 **Inverse function.** Two functions, $y = h(x)$ and $x = g(y)$ are said to be inverses when

230 $g(h(x)) = x$ and $h(g(y)) = y$. The function inverse to $f(x)$ is denoted $f^{-1}(x)$.

231 **Irrational number.** A real number that cannot be expressed as a quotient of two

232 integers, e.g., $\sqrt{2}$. It can be shown that a number is irrational if and only if it cannot be

233 written as a repeating or terminating decimal.

234 **Law of Cosines.** An equation relating the cosine of an interior angle and the lengths of

235 the sides of a triangle. (MW)

236 **Law of Sines.** Equations relating the sines of the interior angles of a triangle and the

237 corresponding opposite sides. (MW)

238 **Line plot**. A method of visually displaying a distribution of data values where each data

239 value is shown as a dot or mark above a number line. Also known as a dot plot. (DPI)

240 **Linear association.** Two variables have a linear association if a scatter plot of the data

241 can be well-approximated by a line. See correlation.

242 **Linear equation.** Any equation that can be written in the form $Ax + By + C = 0$ where A

243 and B are not both 0. The graph of such an equation is a line.

244 **Linear function.** A function, f which may be brought into the form $f(x) = mx + b$. For

245 example $f(t) = 2(t-7)$ represents a linear function.

246 **Line of symmetry** A line that divides a figure into two congruent parts, so that the

247 reflection of either part across the line maps precisely onto the other part.

248 **Logarithm.** The exponent that indicates the power to which a base number is raised to

249 produce a given number. *For example, the logarithm of 100 to the base 10 is 2.* (M)

250 **Logarithmic function.** A function $f(x) = \log_b(x)$ which is inverse to the function $g(x) = b^x$.

251 **Matrix (pl. matrices).** A rectangular array of numbers or variables.

252 **Mean**. A measure of central tendency in a set of numerical data, computed by adding

253 the values in a list and then dividing by the number of values in the list.[2] *Example: For*

254 *the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.*

255 **Mean absolute deviation**. A measure of variation in a set of numerical data, computed

256 by adding the distances between each data value and the mean, then dividing by the

257 number of data values. *Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120},*

258 *the mean absolute deviation is 19.9.*

259 **Measure of variability.** A determination of how much the performance of a group

260 deviates from the mean or median, the most frequently used measure is standard

261 deviation.

262 **Median**. A measure of central tendency in a set of numerical data. The median of a list

263 of values is the value appearing at the center of a sorted version of the list; or the mean

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[2] To be more precise, this defines the *arithmetic mean*.

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264   of the two central values, if the list contains an even number of values. *Example: For the*

265   *data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.*

266   **Midline.** In the graph of a sine or cosine function, the horizontal line halfway between its

267   maximum and minimum values.

268   **Model.** A mathematical representation (e.g., number, graph, matrix, equation(s),

269   geometric figure) for real-world or mathematical objects, properties, actions, or

270   relationships. (DPI)

271   **Modulus of a complex number**. The distance between a complex number and the

272   origin on the complex plane. The modulus of *a + bi* is written $|a + bi|$, and computed as

273    $|a + bi| = \sqrt{a^2 + b^2}$. For a complex number in polar form, $r(\cos\theta + i \sin\theta)$, the modulus is $|r|$

274   .

275   **Multiplication and division within 100**. Multiplication or division of two whole numbers

276   with whole number answers, and with product or dividend in the range 0–100. *Example:*

277   *72 ÷ 8 = 9.*

278   **Multiplicative inverses**. Two numbers whose product is 1 are multiplicative inverses of

279   one another. Example: $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because

280    $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$. (CCSSM)

281   **Network.** a) A figure consisting of vertices and edges that shows how objects are

282   connected, b) A collection of points (vertices), with certain connections (edges) between

283   them.

284   **Non-linear association.** The relationship between two variables is nonlinear if the

285   value of the each variable changes with the value of the other; but the change in one is

286   not simply proportional to the change in the other variable.

287   **Number line diagram.** A diagram of the number line used to represent numbers and

288   support reasoning about them. In a number line diagram for measurement quantities,

289   the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

290   **Numeral.** A symbol or mark used to represent a number.

291   **Order of Operations.** Convention adopted to perform mathematical operations in a

292   consistent order. 1. Perform all operations inside parentheses, brackets, and/or above

293   and below a fraction bar in the order specified in steps 3 and 4; 2. Find the value of any

294   powers or roots; 3. Multiply and divide from left to right; 4. Add and subtract from left to

295   right. (NCTM)

296   **Ordinal number.** A number designating the place (as first, second, or third) occupied

297   by an item in an ordered sequence. (M)

298   **Partition.** The process or result of dividing an object, set of objects, or a number into

299   non-overlapping parts.

300   **Partitive division** (or fair-share division). A division that determines how many are in

301   each group when some quantity is portioned equally into groups. For example, we use

302   partitive division to determine how many pencils each child gets if a parent divides a

303   dozen pencils equally among three children. We can accomplish the calculation with

304   counters by parceling 12 counters into 3 piles ("One for Adam, one for Beth, one for

305   Charlie; two for Adam, two for Beth, two for Charlie…") and checking how many

306   counters are in each pile.

307   **Pascal's triangle.** A triangular arrangement of numbers in which each row starts and

308   ends with 1, and each other number is the sum of the two numbers above it. (H)

309   **Percent rate of change.** A rate of change expressed as a percent. *Example: if a*

310   *population grows from 50 to 55 in a year, it grows by* $\dfrac{5}{50}$ *= 10% per year.*

311   **Periodic phenomena.** Events that recur at regular fixed intervals, for example the

312   solstices.

313   **Picture graph.** A graph that uses pictures to show and compare information. Also

314   known as a pictograph.

315   **Piecewise-defined function.** A function defined by multiple subfunctions, each

316   subfunction applies to a certain interval of the main function's domain.

317   **Polar form.** The polar form of the complex number *a+bi* is any of the following

318   forms: $r\cos\theta + r\,i\sin\theta$, $r(\cos\theta + i\sin\theta)$, (which is often simplified to $r$cis θ), when (r, θ)

319   are polar coordinates of *a+bi* on the complex plane. In any of these forms, $|r|$ is called

320   the modulus and θ is called the argument.

321   **Polynomial.** The sum or difference of terms which have variables raised to nonnegative

322   integer powers and which have coefficients that may be real or complex. The following

323   are all polynomials: $5x^3 - 2x^2 + x - 13$, $x^2y^3 + xy$, and $(1 + i)a^2 + ib^2$. (MW)

324   **Polynomial function.** Any function whose values are determined by evaluating a

325   polynomial.

326   **Prime factorization.** A number written as the product of all its prime factors. (H)

327   **Prime number.** A whole with exactly two distinct factors, 1 and itself.

328   **Probability distribution.** The set of possible values of a random variable with a

329   probability assigned to each.

330   **Properties of equality**. See Table 4 in this Glossary.

331   **Properties of inequality**. See Table 5 in this Glossary.

332   **Properties of integers.** See Tables 3, 4, and 5 in this Glossary.

333   **Properties of operations**. See Table 3 in this Glossary.

334   **Probability**. A number between 0 and 1 used to quantify likelihood for processes that

335   have uncertain outcomes (such as tossing a coin, selecting a person at random from a

336   group of people, tossing a ball at a target, testing for a medical condition).

337   **Probability model.** A probability model is used to assign probabilities to outcomes of a

338   chance process by examining the nature of the process. The set of all outcomes is

339   called the sample space, and their probabilities sum to 1. *See also: **uniform***

340   ***probability model***.

341   **Proof.** A method of constructing a valid argument, using deductive reasoning.

342   **Proportion. (a)** Another term for a fraction of a whole. For example, the "proportion of

343   the population that prefers product *A"* might be 60%. **(b)** An equation that states that

344   two ratios are equivalent, e.g., $\dfrac{4}{8} = \dfrac{1}{2}$ or $4 : 8 = 1 : 2$.

345   **Proportional relationship.** A collection of pairs of numbers that are in equivalent ratios.

346   A ratio A:B determines a proportional relationship, namely the collection of pairs (cA,

347   cB), for c positive. A proportional relationship is described by an equation of the form

348    $y = kx$, where k is a positive constant, often called a constant of proportionality. (P.RPR)

349   **Pythagorean Theorem.** For any right triangle, the sum of the squares of the measures

350   of the legs equals the square of the measure of the hypotenuse.

351   **Quadratic equation.** An equation that includes second degree (and possibly lower

352   degree) polynomials. Some examples are $y = 3x^2 - 5x^2 + 1$, $x^2 + 5xy + y^2 = 1$, and

353   $1.6a^2 + 5.9a - 3.14 = 0$.

354   **Quadratic expression.** A polynomial expression that contains a term of degree 2, but

355   no term of higher degree.

356   **Quadratic function.** A function that can be represented by an equation of the form $y =$

357   $ax^2 + bx + c$, where a, b, and c are arbitrary, but fixed, numbers and $a \neq 0$. The graph of

358   this function is a parabola. (DPI)

359   **Quadratic polynomial.** A polynomial where the highest degree of any of its terms is 2.

360   **Quotitive division** (or measurement division). A division that determines how many

361   equal–size groups can be formed from a given quantity. For example, we use quotitive

362   division to determine how many pies we can buy for $12 when each pie costs $3. We

363   can accomplish the calculation with counters by parceling 12 counters into piles of size

364   3 each ("One, two, three for the first pie; one, two, three for the second pie,…") and

365   checking how many piles there are.

366   **Radical.** The $\sqrt{}$ symbol, which is used to indicate square roots or nth roots. (MW)

367   **Random sampling.** A smaller group of people or objects chosen from a larger group or

368   population by a process giving equal chance of selection to all possible people or

369   objects, and all possible subsets of the same size. (H)

370   **Random variable.** An assignment of a numerical value to each outcome in a sample

371   space. (M)

372   **Range (of a set of data).** The numerical difference between the largest and smallest

373   values in a set of data. (DPI)

374   **Rate.** A *rate* associated with the ratio A:B is $\dfrac{A}{B}$ units of the first quantity per 1 unit of the

375   second quantity. (Note that the two quantities may have different units.) (P.RPR)

376   **Ratio.** A multiplicative comparison of two numbers or quantities, e.g., 4 to 7 or 4 : 7 or

377   $\dfrac{4}{7}$.

378   **Rational expression.** A quotient of two polynomials with a non-zero denominator.

379   **Rational number**. A number expressible in the form $\frac{a}{b}$, where a and b are integers, and

380   $b \neq 0$. See Illustration 1 in this Glossary.

381 **Real number.** A number that corresponds to a point on a number line. See Illustration 1

382 in this Glossary.

383 **Rectangular array.** An arrangement of mathematical elements into rows and columns.

384 **Rectilinear figure.** A planar figure all of whose sides are straight line segments.

385 **Recursive pattern or sequence.** A pattern or sequence wherein each successive term

386 can be computed from some or all of the preceding terms by an algorithmic procedure.

387 **Reflection.** A type of transformation that flips points about a line or a point.

388 **Relative frequency.** The empirical counterpart of a probability. If an event occurs N'

389 times in N trials, its relative frequency is $\dfrac{N'}{N}$. (M)

390 **Remainder Theorem.** If $f(x)$ is a polynomial in x then the remainder on dividing $f(x)$ by x

391 $- a$ is $f(a)$. (M)

392 **Repeated reasoning**. The reasoning involved in solving one mathematical problem that

393 is used again in a different mathematical problem or problems. Mathematically proficient

394 students notice if calculations are repeated, and look both for general methods and for

395 shortcuts… As they work to solve a problem, mathematically proficient students

396 maintain oversight of the process, while attending to the details. They continually

397 evaluate the reasonableness of their intermediate results. (CCSSM) See Standards for

398 Mathematical Practice 8.

399 **Repeating decimal**. A decimal expansion of a number in which, after a certain point, a

400 particular digit or sequence of digits repeats itself indefinitely; the decimal form of a

401 rational number. (M) *See also: **terminating decimal**.*

402 **Rigid motion**. A transformation of points in space consisting of a sequence of one or

403 more translations, reflections, and/or rotations. Rigid motions preserve distances and

404 angle measures.

405 **Rotation.** A type of transformation that turns a figure about a fixed point, called the

406 *center of rotation*.

407 **SAS congruence.** (Side-angle-side congruence.) When two triangles have both two

408 pairs of corresponding sides and also the included angle formed by those sides all to be

409 congruent, then the triangles are congruent.

410 **SSS congruence. (**Side-side-side congruence.) When two triangles have all three pairs

411 of corresponding sides congruent, then the triangles are congruent.

412 **Sample space**. In a probability model for a random process, a list of the individual

413 outcomes that are to be considered.

414 **Scatter plot**. A graph in the coordinate plane representing a set of bivariate data. *For*

415 *example, the heights and weights of a group of people could be displayed on a scatter*

416 *plot.* (DPI)

417 **Scientific notation.** A widely used system in which numbers are expressed as products

418 consisting of a number between 1 and 10 multiplied by an appropriate power of 10, e.g.,

419 $562 = 5.62 \times 10^2$. (MW)

420 **Sequence, progression.** An ordered set of elements, e.g., 1, 3, 9, 27, 81. In this

421 sequence, 1 is the *first term*, 3 is the *second term*, 9 is the *third term*, and so on.

422 **Similarity transformation.** A transformation that can be represented as a rigid motion

423 followed by a dilation.

424 **Simultaneous equations.** Two or more equations containing common variables. (MW)

425 **Sine.** The trigonometric function that for an acute angle of a right triangle is the ratio

426 between the leg opposite the angle and the hypotenuse.

427 **Strategy.** A plan of action designed to achieve a long-term or overall aim. An approach

428 to teaching and learning. (O)

429 **Subitize.** To immediately, and without counting, perceive a quantity.

430 **Tangent. (a)** A line passing perpendicular to a radius at the point lying on the circle is

431 said to be tangent to the circle. **(b)** The trigonometric function that, for an acute angle of

432 a right triangle, is the ratio between the leg opposite the angle and the leg adjacent to

433 the angle.

434 **Tape diagram.** A drawing that looks like a segment of tape, used to illustrate number

435 relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

436 **Terminating decimal.** A decimal number is called terminating if it is a repeating

437 decimal and its repeating digit is 0. *See also: **repeating decimal***.

438 **Third quartile**. For a data set with median *M*, the third quartile is the median of the data

439 values greater than *M. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120},*

440 *the third quartile is 15. See also: **median, first quartile, interquartile range**.*

441 **Transformation.** A prescription, or rule, that sets up a one-to-one correspondence
442 between the points in a geometric object (the *pre-image*) and the points in another
443 geometric object (the *image*). Reflections, rotations, translations, and dilations are
444 particular examples of transformations.

445 **Transitivity principle for indirect measurement.** If the length of object A is greater
446 than the length of object B, and the length of object B is greater than the length of object
447 C, then the length of object A is greater than the length of object C. This principle
448 applies to measurement of other attributes, such as time, weight, area, and volume.

449 **Translation.** A type of transformation that moves every point in a graph or geometric
450 figure by the same distance in the same direction without a change in orientation or
451 size. (MW)

452 **Trigonometric function.** Any of the six functions (sine, cosine, tangent, cotangent,
453 secant, cosecant) that for an acute angle of a right triangle may be expressed in terms
454 of ratios of sides of the right triangle.

455 **Trigonometry.** The study of triangles and trigonometric functions.

456 **Uniform probability model**. A probability model which assigns equal probability to all
457 outcomes. (CCSSM) *See also: **probability model**.*

458 **Unit fraction.** A fraction with a numerator of 1, such as $\frac{1}{3}$ or $\frac{1}{5}$.

459 **Unit rate.** The numerical part of the rate; the word "unit" in "unit rate" is used to highlight
460 the 1 in "per 1 unit" of the second quantity. For example, if 3 melons cost $4.50, then
461 the unit rate is simply the number 4.5/3 =1.5 (without units). (P.RPR)

462 **Univariate.** Relating to a single variable.

463 **Valid. (a)** Well-grounded or justifiable; being at once relevant and meaningful, e.g., a
464 *valid* theory; **(b)** Logically correct. (MW)

465 **Variable.** A quantity that can change or that may take on different values. Refers to the
466 letter or symbol representing such a quantity in an expression, equation, inequality, or
467 matrix. (MW)

468 **Vector.** A quantity with magnitude and direction in the plane or in space, defined by an
469 ordered pair or triple of real numbers.

470 **Visual fraction model.** A tape diagram, number line diagram, or area model.

471 **Whole numbers**. The numbers 0, 1, 2, 3, … . See Illustration 1 in this Glossary.

472

473   **Tables and Illustrations of Key Mathematical Properties,**

474   **Rules, and Number Sets**

475

476   **TABLE 1**. Common addition and subtraction situations.[3]

	Result Unknown	**Change Unknown**	**Start Unknown**
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$

	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[4]
Put Together/ Take Apart[5]	Three red apples and two green apples are on the table. How many apples are on the table?	Five apples are on the table. Three are red and the rest are green. How many apples are	Grandma has five flowers. How many can she put in her red vase and how many in her

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[3] Adapted from Boxes 2–4 of *Mathematics Learning in Early Childhood*, National Research Council (2009, pp. 32–33).

[4] These take-apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[5] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

The *Mathematics Framework* was adopted by the California State Board of Education on November 6, 2013. The *Mathematics Framework* has not been edited for publication.

	$3 + 2 = ?$	green? $3 + ? = 5,\ 5 - 3 = ?$	blue vase? $5 = 0 + 5,\ 5 = 5 + 0$ $5 = 1 + 4,\ 5 = 4 + 1$ $5 = 2 + 3,\ 5 = 3 + 2$

	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[6]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5,\ 5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?,\ 3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?,\ ? + 3 = 5$

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[6] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

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479 **TABLE 2**. Common multiplication and division situations.[7]

480

	Unknown Product $3 \times 6 = ?$	**Group Size Unknown** ("How many in each group?" Division) $3 \times ? = 18$ and $18 \div 3 = ?$	**Number of Groups Unknown** ("How many groups?" Division) $? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[8] Area[9]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?

[7] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[8] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[9] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

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Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

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485   **TABLE 3**. The properties of operations.

486

487   Here *a*, *b*, and *c* stand for arbitrary numbers in a given number system. The

488   properties of operations apply to the rational number system, the real number

489   system, and the complex number system.

490

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every *a* there exists $-a$ so that $a + (-a) = (-a) + a = 0.$
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $\frac{1}{a}$ so that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1.$
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

491

492

493   **TABLE 4**. The properties of equality.

494

495   Here a, b, and c stand for arbitrary numbers in the rational, real, or complex

496   number systems.

497

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

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500

501   **TABLE 5**. The properties of inequality.

502

503   Here a, b, and c stand for arbitrary numbers in the rational or real number

504   systems.

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Exactly one of the following is true: $a < b$, $a = b$, $a > b$.

If $a > b$ and $b > c$ then $a > c$.

If $a > b$, then $b < a$.

If $a > b$, then $-a < -b$.

If $a > b$, then $a \pm c > b \pm c$.

If $a > b$ and $c > 0$, then $a \times c > b \times c$.

If $a > b$ and $c < 0$, then $a \times c < b \times c$.

If $a > b$ and $c > 0$, then $a \div c > b \div c$.

If $a > b$ and $c < 0$, then $a \div c < b \div c$.

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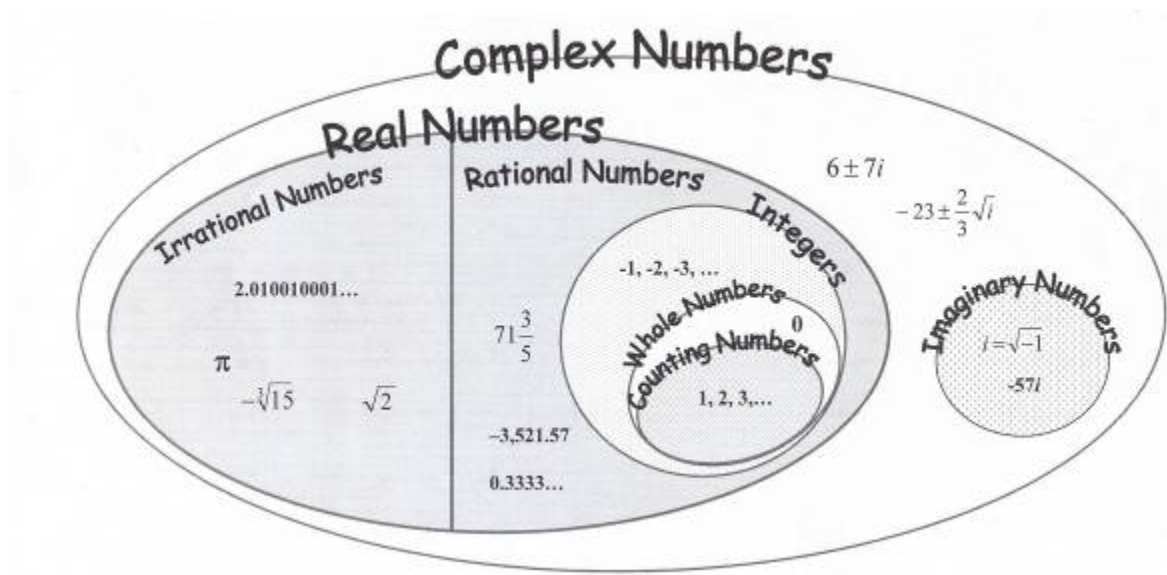511   **ILLUSTRATION 1**. The Number System.

512

513   The Number System is comprised of number sets beginning with the Counting

514   Numbers and culminating in the more complete Complex Numbers. The name of

515   each set is written on the boundary of the set, indicating that each increasing

516   oval encompasses the sets contained within. Note that the Real Number Set is

517   comprised of two parts: Rational Numbers and Irrational Numbers.

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525 **Glossary Sources**

526 **(CCSSM)** http://www.corestandards.org/Math

527 **(DPI)** http://dpi.wi.gov/standards/mathglos.html

528 **(H)** http://www.hbschool.com/glossary/math2/

529 **(K)** Kern, Willis F, and James R. Bland. *Solid Mensuration with Proofs.* 2nd ed.

530 New York: Wiley, 1948. 26.

531 **(M)** http://www.merriam-webster.com/

532 **(MW)** http://www.mathwords.com

533 **(NCTM)** http://www.nctm.org

534 **(O)** http://oxforddictionaries.com/

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536 **(P.RPR)** Draft 6–7 Progression on Ratio and Proportional

537 Relationships http://ime.math.arizona.edu/progressions/

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541

542 *Massachusetts Curriculum Framework for Mathematics*, March

543 2011. http://www.doe.mass.edu/frameworks/math/0311.pdf

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