Calculus[1]

When taught in high school, calculus should be presented with the same level of depth and rigor as are entry-level college and university calculus courses. These standards outline a complete college curriculum in one variable calculus. Many high school programs may have insufficient time to cover all of the following content in a typical academic year. For example, some districts may treat differential equations lightly and spend substantial time on infinite sequences and series. Others may do the opposite. Consideration of the College Board[2] syllabi for the Calculus AB and Calculus BC sections of the Advanced Placement Examination in Mathematics may be helpful in making curricular decisions. Calculus is a widely applied area of mathematics and involves a beautiful intrinsic theory. Students mastering this content will be exposed to both aspects of the subject.

1.0 Students demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity. Students know the definition of convergence and divergence of a function as the domain variable approaches either a number or infinity:

1.1 Students prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions.

1.2 Students use graphical calculators to verify and estimate limits.

1.3 Students prove and use special limits, such as the limits of (sin(x))/x and (1-cos(x))/x as x tends to 0.

Example:

[1] This chapter is taken from the 2005 framework. The standards in this course were first adopted in 1997 and were unchanged in the 2010 adoption of the CA CCSSM.
[2] AP Course Descriptions are updated regularly. Please visit AP Central® (apcentral.collegeboard.com) to determine whether a more recent Course Description is available.

Evaluate the following limits, justifying each step:

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}$$

$$\lim_{x\to 0}\frac{1-\cos(2x)}{\sin(3x)}$$

$$\lim_{x\to\infty}\left(x-\sqrt{x^2-x}\right)$$

2.0 Students demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.

Example:

For what values of x is the function f(x) = x2 – 1/x2 -4x + 3 continuous? Explain.

3.0 Students demonstrate an understanding and the application of the intermediate value theorem and the extreme value theorem.

4.0 Students demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability:

4.1 Students demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function.

4.2 Students demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Students can use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function.

4.3 Students understand the relation between differentiability and continuity.

4.4 Students derive derivative formulas and use them to find the derivatives of algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions.

Example:

Find all points on the graph of $f(x)=\frac{x^2-2}{x+1}$ where the tangent line is parallel to the tangent line at $x=1$.

5.0 Students know the chain rule and its proof and applications to the calculation of the derivative of a variety of composite functions.

6.0 Students find the derivatives of parametrically defined functions and use implicit differentiation in a wide variety of problems in physics, chemistry, economics, and so forth.

Example:

For the curve given by the equation $\sqrt{x}+\sqrt{y}=4$, use implicit differentiation to find $\frac{d^2y}{dx^2}$.

7.0 Students compute derivatives of higher orders.

8.0 Students know and can apply Rolle's theorem, the mean value theorem, and L'Hôpital's rule.

9.0 Students use differentiation to sketch, by hand, graphs of functions. They can identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.

10.0 Students know Newton's method for approximating the zeros of a function.

11.0 Students use differentiation to solve optimization (maximum-minimum problems) in a variety of pure and applied contexts.

Example

A man in a boat is 24 miles from a straight shore and wishes to reach a point 20 miles down shore. He can travel 5 miles per hour in the boat and 13 miles per hour on land. Find the minimal travel time for him to reach his destination and where along the shore he should land the boat to arrive as soon as possible.

12.0 Students use differentiation to solve related rate problems in a variety of pure and applied contexts.

13.0 Students know the definition of the definite integral by using Riemann sums. They use this definition to approximate integrals.

Example:

The following is a Riemann sum that approximates the area under the graph of a function $f(x)$, between $x = a$ and $x = b$. Determine a possible formula for the function $f(x)$ and for the values of a and b: $\sum_{i=1}^{n} \frac{2}{n} e^{1+\frac{2i}{n}}$.

14.0 Students apply the definition of the integral to model problems in physics, economics, and so forth, obtaining results in terms of integrals.

15.0 Students demonstrate knowledge and proof of the fundamental theorem of calculus and use it to interpret integrals as antiderivatives.

Example:

If $f(x) = \int_1^x \sqrt{1+t^3}\,dt$, find $f'(2)$.

16.0 Students use definite integrals in problems involving area, velocity, acceleration, volume of a solid, area of a surface of revolution, length of a curve, and work.

17.0 Students compute, by hand, the integrals of a wide variety of functions by using techniques of integration, such as substitution, integration by parts, and trigonometric substitution. They can also combine these techniques when appropriate.

Example:

Evaluate the following:

$$\int \frac{\sin\left(1-\sqrt{x}\right)}{\sqrt{x}}\,dx \qquad \int_1^e \frac{\ln x}{\sqrt{x}}\,dx \qquad \int_0^1 \sqrt{1+\sqrt{x}}\,dx$$

$$\int \arctan x\,dx \qquad \int \frac{\sqrt{x^2-1}}{x^3}\,dx \qquad \int \frac{dx}{e^x\sqrt{1-e^{2x}}}$$

18.0 Students know the definitions and properties of inverse trigonometric functions and the expression of these functions as indefinite integrals.

19.0 Students compute, by hand, the integrals of rational functions by combining the techniques in standard 17.0 with the algebraic techniques of partial fractions and completing the square.

20.0 Students compute the integrals of trigonometric functions by using the techniques noted above.

21.0 Students understand the algorithms involved in Simpson's rule and Newton's method. They use calculators or computers or both to approximate integrals numerically.

22.0 Students understand improper integrals as limits of definite integrals.

23.0 Students demonstrate an understanding of the definitions of convergence and divergence of sequences and series of real numbers. By using such tests as the comparison test, ratio test, and alternate series test, they can determine whether a series converges.

Example:

Determine whether the following alternating series converge absolutely, converge conditionally, or diverge:

$$\sum_{n=3}^{\infty}(-1)^n\left(\frac{2^n}{n!}\right) \qquad \sum_{n=3}^{\infty}\frac{(-1)^n}{n\ln n} \qquad \sum_{n=3}^{\infty}(-1)^n\left(\frac{1+n}{n+\ln n}\right)$$

24.0 Students understand and can compute the radius (interval) of the convergence of power series.

25.0 Students differentiate and integrate the terms of a power series in order to form new series from known ones.

26.0 Students calculate Taylor polynomials and Taylor series of basic functions, including the remainder term.

27.0 Students know the techniques of solution of selected elementary differential equations and their applications to a wide variety of situations, including growth-and-decay problems.